Using Equity Options to Imply Credit Information

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Abstract

The evolution of credit derivatives has inspired many researchers to investigate the behaviour of credit spreads. Today the growing consensus is that the equity option market provides sufficient information to estimate latent credit parameters. Hull, Nelken and White (2005) propose a clever approach to estimate credit spreads from the equity option market. In this paper we first perform a time series analysis to test the conjecture of an existing relationship between credit spreads and implied equity volatility and find strong evidence of a positive relationship. We also propose an extension to Hull et al.'s paper that significantly improves credit spread estimation.

Keywords: credit spreads, implied volatility, Merton's model, liquidity, first-passage, default probability

I. Introduction

Since the early 1990's the study of credit derivatives has evolved rapidly throughout the financial industry. In order to accommodate the evolving credit derivatives market, the Basel Committee established requirements instructing banks to create internal systems that measure and manage their credit risk objectively. These requirements have made the challenging task of understanding and modeling credit behaviour vital and much research continues to be devoted to this subject. Credit models can be classified into two main categories:

i. Structural Models pioneered by Merton (1974)
   The underlying idea being that defaults can be predicted by equity markets

ii. Reduced-Form Models originally studied by Jarrow and Turnbull (1995)
   The underlying idea being that defaults occur at random

Another category of models recently suggested includes hybrid models. This is investigated in the work of Cathcart et al. (1999) who propose a hybrid of both reduced-form and structural models and Chen & Panjer (2003) who show that by applying a jump-diffusion process the yield spreads between structural and reduced-form models can be unified.

Structural models are praised for their intuitive nature and useful capabilities of providing a valuable link between credit and equity markets. Understanding the relationship between the equity and credit markets will prove profitable for financial institutions awarding them the capability to price credit products accurately, effectively hedge credit and equity exposure and innovate hybrid products such as the Equity Default Swap discussed in Medova & Smith (2004). For years academics have suggested the existence of a relationship between credit and equity and today there continues to be growing empirical evidence supporting their connection. The demise of Enron is case in point. Even though Enron's stock price had been declining for a considerable period of time, it was rated investment grade up until one month before it filed for bankruptcy. After finally being downgraded to junk bond status, it filed for bankruptcy within four days, providing no time for lenders to take appropriate action; lenders could have minimized their losses had they taken note of the declining stock price and related it to the company's credit quality. The credit-equity relationship is further supported in the work of Hull et al. (2004) who study the delayed credit reaction to equity movements.

The current research trend continues to support the use of equity information to predict unobservable credit parameters. Many researchers such as Zou (2003), Medova & Smith, and Finger (2002), have concluded that equity information is sufficient to infer values for unknown credit parameters. While it is obvious that equity markets are better predictors than vague financial statements alone, the growing consensus among practioners as well as academics, is that by applying information from the equity option markets result in greater precision and accuracy when estimating latent credit parameters. This concept is investigated in the work of Jackwerth & Rubinstein (1996), Zou (2003), Hull et al. (2005) and Finger (2005). In their work Hull et al.(2005) introduce a method to calculate credit spreads using option information via a Merton-style model. We will henceforth refer to their approach as "The Alternative Model". While this model is intuitive and coherent, it is not without limitations. In this paper we examine how to alleviate some of the model's weaknesses by relaxing a one time step assumption; we propose applying a first-passage calculation and look at the effects of different barriers. Furthermore we re-define equity from being a call option to being a compound option and provide closed form solutions for all definitions and parameters. We refer to our extension as "The First-Passage Alternative (FPA) Model" henceforth.

This paper is divided into six sections. In the next section structural models are reviewed and differences between models are compared. In the third section we carry out a time series analysis to validate the notion that a relationship between credit spreads and implied equity volatility exists. The fourth section briefly reviews the Alternative Model and discusses its limitations. In the fifth section we introduce, analyze and assess the performance of the FPA model; finally we draw conclusions in the sixth section.

II. Structural Models

Even though structural models are generally criticized for their production of low implied credit spreads (especially for short-term spreads) and downward term structure[1], they continue to gain popularity throughout the financial industry due to their ease of implementation and intuitive nature. Widely used commercial applications such as Moody's KMV (a credit analytics firm acquired by Moody's), and CreditGrades™ (a joint venture of Deutsche, Goldman Sachs, JP Morgan and RiskMetrics) adopt a structural framework.

A. Structural Models and Different Extensions

Structural models were pioneered by Merton (1974) who proposed that assets follow a log-normal process with constant drift and volatility. Assuming a simple capital structure, assets of a firm can be comprised of equity and debt. Equity can be thought of as a call option on the firms assets struck at the debt face value, and debt can be regarded as a zero coupon bond. Default occurs at debt maturity in the event that a firm has insufficient funds to pay back its bondholders (occurring when the debt face value exceeds the asset value) at which time the firm liquidates all its equity to the bondholders[2]. A first-passage approach relaxes Merton's assumption that default can occur only at debt maturity, rather this calculation assumes that default can occur at any time up to debt maturity when a chosen barrier is triggered by the asset process. Black & Cox (1976) were the first researchers to include a first-passage calculation, this calculation can therefore be used to relax Merton's one-time step default measure. Since then numerous researchers have taken strides to make further extensions, for example Geske (1977) models the coupon on the bond as a compound option[3]. As the coupon dates arrive the shareholders decide whether the firm can pay the coupon, if so the firm survives. If however the coupon payment exceeds the firms assets, default occurs and bondholders receive the firm in its entirety. In the Longstaff-Schwartz (1995) model, interest rates are assumed to be stochastic. Default occurs when the firm's asset value declines to a particular barrier level resulting in bondholders recovering a constant fraction of the debt value. Leland & Toft (1996) assume the firm issues constant debt continuously with a fixed maturity paying continuous coupons. If equity holders cannot pay bondholders then default occurs and equity holders get nothing while bondholders receive some fraction of the asset value. The Collin-Dufresne & Goldstein (2001) model extends the Longstaff-Schwartz model as it tries to correct the problem of small credit spreads and incorrect term structure by incorporating a constant leverage.

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[1] Many researchers such as Sarig & Wara (1989), Fons (1994), He et al. (2000), Giesecke (2003) have found evidence of a downward credit term structure.

[2] This risk neutral default probability can be interpreted using Black&Scholes equations with the condition that the option is not exercised (Nielsen(1993)).

[3] This model is not practical for compound options with many coupon dates as it is difficult to express in simple integrals which may result in intensive calculations.

The work of Eom et al. (2003) examines these models and analyzes the implied credit spread produced by each model. They find that the spreads produced using Merton's and Geske's approach are on average too small, while the Longstaff-Schwartz, Leland-Toft, and Collin-Dufresne & Goldstein models generate spreads that on average are too high. This is in agreement with Ogden (1987) and Lyden & Saraniti (2000) who find that Merton's implied credit spreads tend to be underestimated compared to market quotes. Eom et al. do not suggest that structural models are unable to predict sufficiently high spreads, in fact they find the Leland-Toft model overestimates credit risk on shorter maturity bonds. Gemmill (2002) on the other hand finds that Merton's implied credit spreads are on average of similar magnitude to market spreads. In his work he considers between 20 to 78 UK capital structure bonds for monthly periods between February 1992 - April 2001. He finds that market spreads are on average equal to model spreads and notes that the model spreads are small relative to market spreads for bonds near maturity and for bonds with low leverage. He considers a leverage with drift over time and concludes that it generates an increasing term structure for credit spreads. He supports Collin-Dufresene & Goldstein findings that illustrate leverage estimates as having a large influence on implied spreads. Furthermore Gemmill agrees that the difference between market spreads and model spreads is greatly sensitive to the term-structure of interest rates.

B. Barriers

Barriers are used in first-passage calculations and are the key differentiating factor that distinguishes structural models from one another. First-passage default probability equations are based on the Brownian motion running minimum calculation. For example assume that the assets follow a log-normal process:

$$A_T = A_0 e^{(\mu - \frac{1}{2}\sigma^2)T + \sigma W_T} \tag{1}$$

where: A_T is the asset value at time T , A_0 is the asset value at time 0 , μ is the drift, σ is the asset volatility, W_T is a standard Brownian motion

The running minimum of the log-asset process is defined as:

$$M_T = \min_{s \le T} \{(\mu - \frac{1}{2}\sigma^2)s + \sigma W_s\} \qquad (2)$$

Default is defined as the event when the minimum asset value hits the barrier and the default probability measures the probability in which that occurs:

Default Probability (x,T) = P[A_s $\le$ Barrier for some s = 0……T]

$$= P\,[M_T \le x] \qquad \text{with } x = \ln(\frac{Barrier}{A_0}) \qquad (3)$$

It is known that the running minimum of Brownian Motion has an inverse Gaussian distribution (Harrison (1990)). Therefore default probability is given by:

$$\begin{aligned} \mathrm{P}[\mathrm{M_T} \le x] &= \psi(T, x) \\ &= N(\frac{x - mT}{\sigma\sqrt{T}}) + \exp(\frac{2mx}{\sigma^2}) N(\frac{x + mT}{\sigma\sqrt{T}}) \end{aligned} \qquad (4)$$

where: $m = \mu - \frac{1}{2}\sigma^2$ and $x = \ln(\frac{Barrier}{A_0})$

We can see from the above derivation that the choice of barrier is instrumental to the probability of default calculation.

A constant default barrier, usually taken to be a multiple of debt face value, is the simplest form of barriers. While this assumption is perceived to be overly simplistic, it is investigated in the work of Longstaff & Schwartz, Leland & Toft, and Giesecke (2004).

CreditGrades^{TM} , perceived as the industry standard for many financial institutions, has adopted a random barrier methodology. They propose a log-normal barrier with mean set equal to 50% and volatility 30% (Finger (2002)). They correct the problem of low short-term spreads by assuming that the probability of default at t =0 does not equal zero. Zou

(2003) criticizes CreditGrades™ and similar commercial applications of structural models, stating that they are incapable of producing spreads on high quality debt. Zou believes the problem to be in the assumptions regarding log-normal assets and constant asset volatility. In his calculation he assumes a constant recovery rate and describes how to infer a probability density function for assets from equity call options.

Giesecke (2003) also looks at random barriers and tries to solve the problem of under predicted short-term spreads by using a barrier uniformly distributed between zero and the initial asset value[4]. This choice of barrier is intended to represent investors being uninformed about default barriers.

III. The Relationship between Credit Spreads and Implied Equity Volatility

In order to draw concrete conclusions we must first validate the intuition that the equity option market contains fundamental credit information. The purpose of this section is to determine if in fact a relationship between credit spreads and implied equity volatility exists[5], and to propose some preliminary conclusions through time series analysis; thus gaining insight into the association between the credit and equity option markets.

A. Data Description

We ensure a controlled experiment by subjecting our equity data to US exchange traded options in order to eliminate significant external factors such as liquidity issues. Optimally we would like to study the most liquid volatility (at-the-money or 10% out-of the-money) against the most liquid credit spread (five year) and in a perfect world we could plot five year credit spreads against five year at-the-money implied volatilities. In reality this criteria is not feasible as the longest dated exchange traded option is approximately two years in maturity. In order to avoid any presumptions on term and/or moneyness, we examine six month, one year and two year data, hoping that the relationship between credit spreads and implied volatility will converge to the same conclusion. Our analysis consists of comparing six months, one year and two year credit spreads against 0%, 10%, 20%, 30%, 40% and 50% out-of-the-money (OTM) volatility (where 0%OTM implies at-the-money volatility). Since credit spreads do not drastically change day-over-day, we calculate and compare monthly intervals for the period of

[4] In his calculation Giesecke assumes zero recovery

[5] When referring to credit spreads we refer to CDS spreads. All equity data is obtained from Bloomberg

October 2000 to September 2003. To further eliminate any biases regarding credit quality we subject our analysis to four US firms of different credit ratings.

B. Analysis

We commence our analysis by creating a time series of credit spreads and implied volatility for the following US firms: Ford (F), Delphi (DPH), IBM, and Clear Channel Communications (CCU). For each firm we plotted the following:

- Six month credit spread against 0% OTM volatility expiring in six months
- Six month credit spread against 10% OTM volatility expiring in six months
- Six month credit spread against 20% OTM volatility expiring in six months
- Six month credit spread against 30% OTM volatility expiring in six months
- Six month credit spread against 40% OTM volatility expiring in six months
- Six month credit spread against 50% OTM volatility expiring in six months

These sets of graphs were repeated using one-year credit spreads across all implied equity volatilities maturing in one year, and two-year credit spreads across all implied equity volatilities expiring in two years. An example is illustrated in Figure 1 depicting monthly credit spreads for CCU when graphed against at-the-money volatility, 20% OTM, and 50% OTM. The graphs demonstrate a strong relationship between credit spreads and implied volatility regardless of maturity and/or moneyness. For the purpose of completeness we apply statistical tests for assurance, correlation was used as our statistical test of measure. Both parametric correlation (Pearsonian) and non-parametric (Spearman Rank Order)[6] are calculated in order to eliminate any unnecessary assumptions about distributions[7]. Results are shown in Table 1. Both correlation measures simultaneously confirm our intuition, indicating that a strong positive relationship exists between credit spreads and implied equity volatility. According to our sample this relationship holds true for all spreads and all volatility regardless of maturity, credit rating and/or moneyness.

[6] Refer to Hull et al.(2005) for full derivations and description of Spearman's Rank Order.
[7] Pearsonian correlation is used when the underlying distribution is assumed to be normal.

IV. The Alternative Model

Hull et al. (2005) use Merton's classical model and defines equity today (E_0) as a call option on a firms assets (A_0) struck at the face value of the debt (D).

$$E_0 = A_0[N(d_1) - LN(d_2)] \tag{5}$$

where: $d_1 = \frac{-\ln(L)}{\sigma\sqrt{T}} + \frac{1}{2}\sigma\sqrt{T}$, $d_2 = d_1 - \sigma\sqrt{T}$, $N(\bullet)$ is the cumulative standard normal distribution function, L is leverage of a firm defined as: $L = \frac{De^{-rT}}{A_0}$

Since equity is a function of the asset value, application of Ito's Lemma results in the following:

$$E_0\, v = N(d_1)\, A_0\, \sigma \tag{6}$$

where: v is the volatility implied from equity

If the face value of the debt is known, then solving equations (5) and (6) simultaneously provides values for A_0 and σ .

By using information of two equity put options, Hull et al. (2005) suggest a way to imply unobservable credit parameters such as asset volatility and leverage by solving a system of equations. We leave it to the reader to refer to the paper for details and complete derivations, however for the purpose of illustration we re-iterate the system of equations used to solve for the credit parameters.

Using the price for a compound option [Geske (1979)] a put option on a call equates to the following:

$$Put - on - Call = De^{-rT} M(-a_2, d_2; -\sqrt{\tau / T}) - A_0 M(-a_1, d_1; -\sqrt{\tau / T}) + Ke^{-r\tau} N(-a_2) \tag{7}$$

where: K is the put strike price, τ is the put maturity, T is the expiry time of the call, A_τ^* is the critical asset value (the asset value at time τ below which the put on the equity will be exercised), M is the standard cumulative bivariate normal distribution (see Appendix for details),

$$a_1 = \frac{\ln(A_0 / A_\tau^* e^{-r\tau})}{\sigma\sqrt{\tau}} + \frac{1}{2}\sigma\sqrt{\tau} \quad , \; a_2 = a_1 - \sigma\sqrt{\tau}$$

Recalling that Merton expresses equity as a call option, and using the BlackScholes price of the put, the following holds true:

$$\begin{aligned} \textit{Put-on-Call} &= \textit{Put-on-Equity} \\ &= Ke^{-rT} N(-d_2^*) - E_0 N(-d_1^*) \end{aligned} \qquad (8)$$

The Alternative Model makes use of this relationship and expands the equation above to produce:

$$\begin{aligned} LM(-a_2, d_2; -\sqrt{\tau / T}) - M(-a_1, d_1; -\sqrt{\tau / T}) + \kappa N(-a_2)[N(d_1) - LN(d_2)] = \\ [\kappa N(-d_2^*) - N(-d_1^*)][N(d_1) - LN(d_2)] \end{aligned} \qquad (9)$$

where: $d_1^* = \frac{-\ln(\kappa)}{\nu\sqrt{\tau}} + 0.5\nu\sqrt{\tau}$, $d_2^* = d_1^* - \nu\sqrt{\tau}$, ν is the implied volatility of the put on equity, α (a scalar multiple of the forward asset value at which the option is at-the-money) and κ (the ratio of the strike price to the forward equity price) such that: $A_\tau^* = \alpha A_0 e^{r\tau}$ and $K = \kappa E_0 e^{r\tau}$

To obtain another equation in the system, equation (5) is used to determine the implied strike level such that:

$$\kappa = \frac{\alpha N(d_{1,\tau}) - LN(d_{2,\tau})}{N(d_1) - LN(d_2)} \qquad (10)$$

where: $d_{1,\tau} = \frac{-\ln(L/\alpha)}{\sigma\sqrt{T-\tau}} + 0.5\sigma\sqrt{T-\tau}$; $d_{2,\tau} = d_{1,\tau} - \sigma\sqrt{T-\tau}$

After solving equations (6), (9) and (10) simultaneously, values for asset volatility and leverage are obtained which are then used to estimate credit spread.

A. Analysis of The Alternative Model

Because of volatility skews the calculation of credit spreads will depend on which put option is used (ie. options with lower strikes will have higher implied volatility). To avoid liquidity issues we implement the Alternative Model using the most liquid (at-the-money) puts, and compare the calculated monthly credit spreads (Hull et al. (2005))and compared them to market quotes during the period of January 2001 through September 2004. In order to assess this model's performance we subject it to various credit conditions. We examine situations when:

- Credit activity is stable
- Credit activity is volatile
- Firms are of high credit quality
- Firms are of poor credit quality

We select three firms of various credit rating that exhibit different monthly credit shocks throughout the observed time period for our analytical analysis; namely F (volatile credit activity), CCU (stable credit activity), LU (non-investment grade firm which is downgraded).

Figures 2 and 3 illustrate the results for F and CCU when implied credit spreads obtained from the Alternative Model are plotted against market quoted credit spreads[8] and implied equity volatility; correlation measures between market credit spreads and spreads implied from the model are also included. Next we examine the model's performance to non-investment grade firms, we consider LU for a time period where the company was downgraded from B to CCC+ ; in this example we compare daily credit spreads in order to observe the model's behaviour leading up to the event, this is depicted in Figure 4.

The graphs suggest that the Alternative Model underestimates credit spreads. With the exceptions of a few points the model produces close to zero spreads for varying implied volatility. In general we find this model to be insensitive to implied equity volatility of approximately 50% or less; this conclusion is in consensus with Hull et al's (2005) Figure 3. For the readers convenience we have included a copy of Hull et al's Figure 3 in our paper which we refer to as Graph1. These findings suggest that the Alternative Model

[8] These monthly credit spreads are not the exact market quotes for a particular day, rather we graph an average of the last 3 days of market quoted credit spread as its believed to be a better representative.

should only be used for highly distressed scenarios which will coincide with unusually high implied volatility.

B. The Alternative Model's Deficiencies

The model's insensitivity to implied volatility is inconsistent with market trend as well as our previous findings in Section III. Furthermore the almost zero spread is contradictory to the findings of Duffie & Singleton (2003) who suggest that in general the five year credit spread for BB firms should be of the magnitude of two percent and approximately one percent for BBB rated firms. To properly understand the results produced by the Alternative Model, we must address all factors affecting the outcome. Liquidity issues play a significant role in any calculation and quantifying the impact of liquidity is a difficult task. Illiquid options (usually perceived to be anything less than 65% the strike price) do not provide reliable market quotes due to the lack of demand. Therefore in order to eliminate any external biases we subjected our analysis to at-the-money options; which in terms of skewness translates to options with the lowest volatility. Since the calculated credit spread is an increasing function of implied equity volatility, then it is possible that by using out-of-the-money put option information (even though undependable) could result in increased credit spread estimates. Ignoring illiquidity, we investigate the significance of the option skew and decide whether choosing out-of-the-money puts is enough to resolve the issue of underestimated spreads. We consider an investment grade credit firm and estimate monthly credit spreads from January 2001 to October 2001 using out-of-the-money puts expiring in approximately two months. Figure 5 depicts the results when credit spreads are calculated via the Alternative Model using both at-the-money and out-of-the-money puts. The graph implies that the added liquidity risk did not improve results as minimal difference was made when lower strikes were chosen.

Next we address the Alternative Model's insensitivity to implied volatility of less than 50%. This poses a fundamental problem from a practical standpoint as most options are generally priced with an implied volatility between 20%-40%. To illustrate this we examine at-the-money implied volatility of 170 US exchange traded options maturating in approximately two months; we find the average implied volatility of our sample to be 27%. Table 2 illustrates the results. We investigate the possibility that the insensitivity to implied volatility and underestimation of spreads can be a resultant from the model's conservative assumptions such as constant face debt value and default occurring only at debt maturity. Relaxing the former assumption by assuming a non-constant debt (ie. non-constant strike price) is beyond the scope of this paper, therefore we focus our analysis on relaxing the one-time step assumption. In the next section we introduce the First-Passage Alternative Model and study the effects of applying constant, deterministic and random barriers.

V. The First-Passage Alternative (FPA) Model

The FPA Model extends the Alternative Model by including a first-passage methodology, an analogous extension pioneered by Black & Cox (1976). In other words this model assumes default to occur if the assets hit a certain barrier at any time up to debt maturity. We make the assumption that once the company has defaulted, it remains in default at which point the bond holders receive a recovery amount (RR). The recovery amount may be equal to the asset/barrier value or may be determined exogenously. The following table depicts the FPA Model's payoff structure (for simplicity we assume bond and equity holders get paid at debt maturity):

Case	**Bond Holders**	**Equity Holders**
No Default: Assets do not hit barrier at any time	D	A_T-D
Default: Assets hit the barrier at sometime before or at debt maturity	RR	0

A. Estimating Recovery Rate

From the table above we can see that recovery rates play an important role in calculating credit spreads. Due to the scarcity of recovery rate data, determining a precise and reliable measure for recovery rate is improbable which usually results in inaccurate credit spread estimates. Many researchers assume a constant recovery rate exogenously (in practice this is usually assumed to be 30-40% of the debt face value). By defining recovery rate to be the ratio of asset value to debt face value and conditioning on the event of default, Altman (2001) show how recovery rate can be estimated for Merton's Model. Believing the concept of stochastic recovery to be a theoretically sound assumption, we adopt the principles outlined in Altman (2001) and Liu et al. (1997) and extend their derivation to incorporate a first-passage methodology. When referring to the

FPA Model henceforth we are assuming a first-passage approach with stochastic recovery. In addition to recovery rates, credit spread calculations heavily depend on the choice of barriers. We investigate the different influences imposed when using constant, deterministic and random barriers.

B. Using a Constant Barrier

In order to understand the repercussions of the one-time step assumption made in the Alternative Model we begin our analysis by applying a constant barrier (which we equate to the debt face value (D) when calculation credit spreads. Credit spreads can be defined as the additional yield to a zero coupon bond with respect to the risk-free (r).

From the table above it is also assumed that we can also assume that bond holders receive the debt face value or in the event of default some recovery (RR) ; therefore the following can be expressed[9]:

$$\begin{aligned} De^{-(r+CreditSpread(0,T))T} &= e^{-rT}E^{Q}[D\cdot 1_{\{\tau_d>T\}}+RR\cdot 1_{\{\tau_d<T\}}] \\ &= e^{-rT}[D\cdot(1-Default\ probability)+RR\cdot Default\ probability] \qquad (11) \\ CreditSpread(0,T) &= -\frac{1}{T}\ln(1-Default\ probability(1-\frac{RR}{D})) \end{aligned}$$

To assess the performance of this model, we solve the same system of equations proposed by Hull et al. to calculate monthly credit spread for Ford.

It is important to note however that by introducing a first-passage methodology our new calculation of probability implies that equity would need to be re-defined as a down-and-out-call (DOC) option, rather than simple a european call as Hull et al's derive. In theory, we could solve the same system of equations provided in Hull et al. in order to achieve our unknown parameters, as the definition of an equity being a european call option still leads to sound interpretations. However for the purpose of consistency we chose to investigate the effects of re-defining equity as a DOC option which are provided at the end of this paper (see Appendix for derivations). We derive and compare the expanded equations to Hull et al's equation 5, 7, 10 respectively.

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[9] We can use $\psi(T,x)$ as the default probability in (4), by applying Girsanov's Theorem and setting the market price of risk equal to $\frac{\mu-r}{\sigma}$.

We continue our analysis and use the same time period as indicated in Section III in order to draw parallel observations; results using a constant barrier are depicted in Figure 6. Comparing one-time step (Figure 2) and first-passage (Figure 6) calculations we find that little improvement is made between the new probability calculation and the one assumed originally in the Alternative Model. We conclude that a more sophisticated barrier is therefore required.

C. Using a Deterministic Barrier

Next we investigate the effects of a deterministic barrier (which we equate to the present value of the debt face value (De^{-rT})). Under this regime default occurs at any time the asset value falls below the present value of the debt, using equation (4) this defines default probability to be:

$$\textit{Default Probability} = N\left(\frac{\ln(L) - mT}{\sigma\sqrt{T}}\right) + \exp\left(\frac{2m\ln(L)}{\sigma^2}\right)N\left(\frac{\ln(L) + mT}{\sigma\sqrt{T}}\right) \qquad (12)$$

We carry out the previous analysis calculating monthly credit spreads using equation (11) for Ford benchmarking against market quoted spreads, Figure 7 depicts the results. The increasing correlation measure for this calculation suggests minor improvement is made over both the Alternative Model and constant barrier calculation. However the conclusions remain the same, implied credit spreads are still underestimated and insensitive to implied volatility. We suggest that a more complicated barrier is required.

D. Using a Random Barrier

Random barriers are generally more complicated as the exact location of the barrier at any point in time is not known, rather it is estimated using an assumption made on its distribution. We apply Giesecke's logic and hypothesize that investors do not have complete information and make no assumption on default barriers, this translates to a random barrier with uniform distribution. We also note that in theory this choice of barrier should resolve the problem of underestimated short-term model spread, a general drawback incurred by structural models.

We define the asset process as: $A_t = A_0 e^{V_t}$ (13)

where: $V_t = (\mu - \frac{1}{2}\sigma^2)t + \sigma W_t$

We let B_A be the barrier in asset space lying anywhere between 0 and A_0.

$$B_A = A_0 e^{B_v} \tag{14}$$

where: B_v is the corresponding barrier in V_t-space.

Assuming a uniform distribution for B_A on $[0, A_0]$, implies that B_v has a negative exponential distribution with parameter 1, lying anywhere between -∞ and 0. We apply the same principles of the running minimum equation (2) to calculate default probability:

$$\begin{aligned} \text{Default probability } &= P[\min_{\{s \le T\}} A_s < B_A] \\ &= P[M_T < B_v] \\ &= \int_{-\infty}^{0} \psi(t,y) e^{y} dy \end{aligned} \tag{15}$$

Integrating by parts leads to the following closed form solution[10]

$$\int_{-\infty}^{-\nu} \psi(T,y) e^{y+\nu} dy = N(\frac{-\nu - mT}{\sigma\sqrt{T}}) - e^{\nu + s\kappa} N(\frac{-\nu - \upsilon T}{\sigma\sqrt{T}}) + \frac{1}{\gamma} e^{(1-\gamma)\nu} N(\frac{mT - \nu}{\sigma\sqrt{T}}) - \frac{1}{\gamma} e^{\nu + \beta T} N(\frac{\delta T - \nu}{\sigma\sqrt{T}}) \tag{16}$$

where: $\upsilon = m + \sigma^2$, $\kappa = m + \sigma^2/2$, $\gamma = 1 + 2m/\sigma^2$, $\delta = m - \gamma \cdot \sigma^2$, $\beta = -m \cdot \gamma + \gamma^2 \cdot \sigma^2/2$

Again we compare the performance of Ford under this new methodology. Figure 8 illustrates the results and suggests that the use of the random barrier improves results drastically. The correlation measure indicates that this model produces implied credit spreads which adequately captures the magnitude and direction of market quotes, while the graph shows sensitivity to implied equity volatility of all ranges.

[10] Refer to Giesecke (2003) for full derivation.

E. Analysis of the FPA Model

We emulate the analysis conducted in Section IV in order to test the FPA model's performance. We examine the model's ability to estimate market quoted credit spreads as we compare monthly spreads for F, CCU, and LU. We solve Hull et al. proposed system of equations, and in trying to eliminate liquidity issues we use at-the-money puts to imply credit spreads. Figure 8 to 10 illustrate the results for the FPA Model. Fortunately the inclusion of a random barrier proves to be a vast improvement over previous approaches capturing magnitude and sensitivity to implied equity volatility. Recall one major limitation of the previous models including the Alternative Model was their insensitivity to implied equity volatility of less than 50% (Graph 1). To assist us in our comparison we try and replicate Graph 1 plotting at-the-money implied volatilities and corresponding implied credit spreads obtained using the FPA Model. Our sample size includes 90 US exchanged traded firms and the results are depicted in Figure 11. We compare Figure 11 to Hull et al's graph and notice that the FPA estimates deliver much more reasonable results. Credit spreads range from 0%-10% and are produced from implied equity volatilities ranging between 10%-65%, a conservative contrast to values obtained from the Alternative Model.

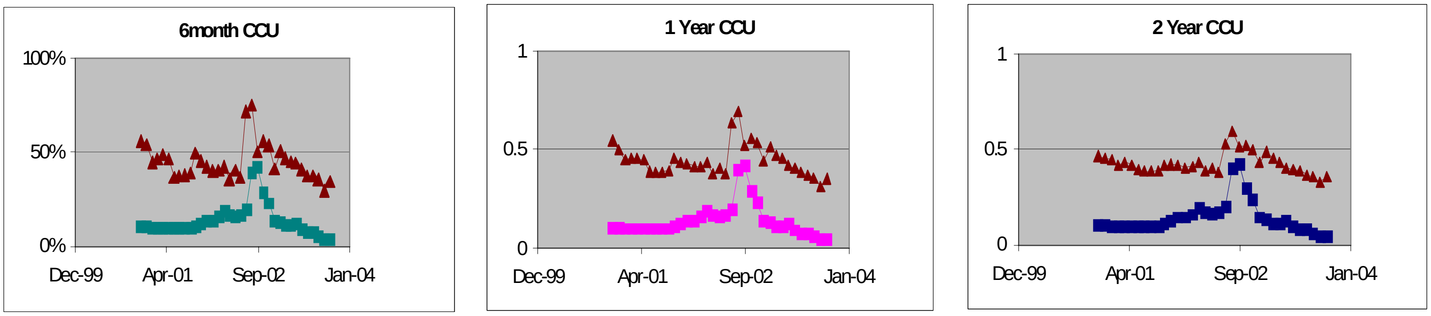
Extending our analysis we include 90 US firms of varying credit qualities calculating monthly spreads for the last four years using the FPA Model as well as the Alternative Model. For each time series we measure the correlation between model spreads and market quotes, and in addition we include the median to shed light on spread magnitude. The results are summarized in Table 3 and indicate that under each scenario the strength of the relationship is significantly stronger between the FPA Model and market quotes. The FPA Model produces a higher correlation which is always positive with market quotes and of correct magnitude. It is evident that by extending the Alternative Model to include a first-passage calculation using a random barrier of uniform distribution improves credit spreads estimation significantly.

VI. Conclusions

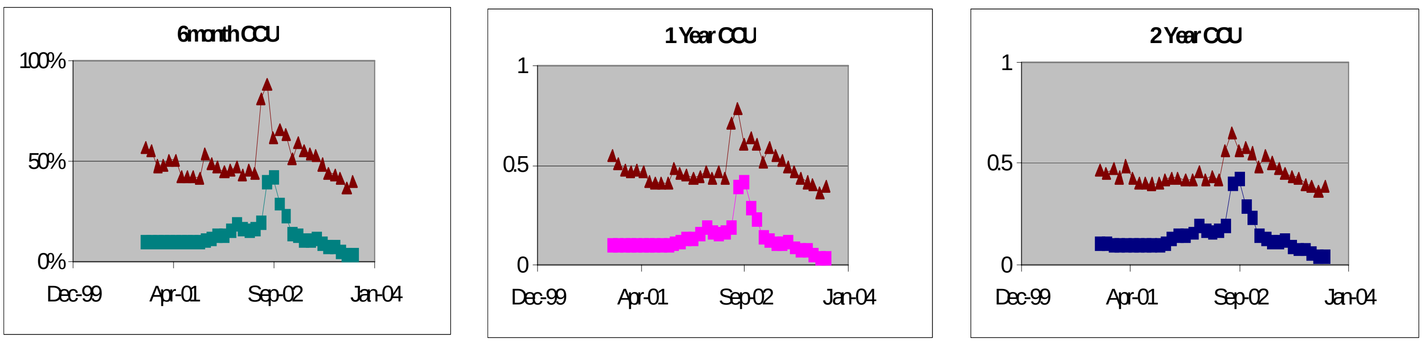
The difficulty in creating a credit model that fits credit spreads lies in the choice of parameters, and in the dependency of required input parameters, which are not easily (if at all) observable, making the task of producing reliable estimations challenging. For a long period of time many academics have suggested that equity markets contain enough information to decipher credit information, however the growing consensus among the industry is that by including equity option information results in substantial enhancements to estimating credit information. We analyze this theory by running a time series analysis between market credit spreads and the corresponding implied equity volatility. We consider different terms and moneyness and find that there exists a strong positive relationship between the credit spreads and implied equity volatility. Hull *et al*. used the classical Merton model to create The Alternative Model, which uses equity option market information to calculate credit spreads. We implement this model and find that the implied credit spreads in general are underestimated and insensitive to implied equity volatility less than 50%. In our analysis we propose an extension of this model (The First-Passage Alternative Model) where we relax a one-time step default probability measure and find that optimal results are obtained when a uniform stochastic barrier is incorporated. We compare the performance of the two models to market quoted spreads and find that the FPA Model produces a much accurate estimation of credit spreads and demonstrates sensitivity to implied volatility of all ranges. One of the attractive features of this model is its ease of implementation, which required over simplified assumptions such as constant interest rates and constant debt face value. A possible next avenue to explore could be to study the effects of including stochastic or deterministic values for interest rates and/or debt values.

Figure 1 : Time Series of 5year Credit Spread and Various Implied Equity Volatility

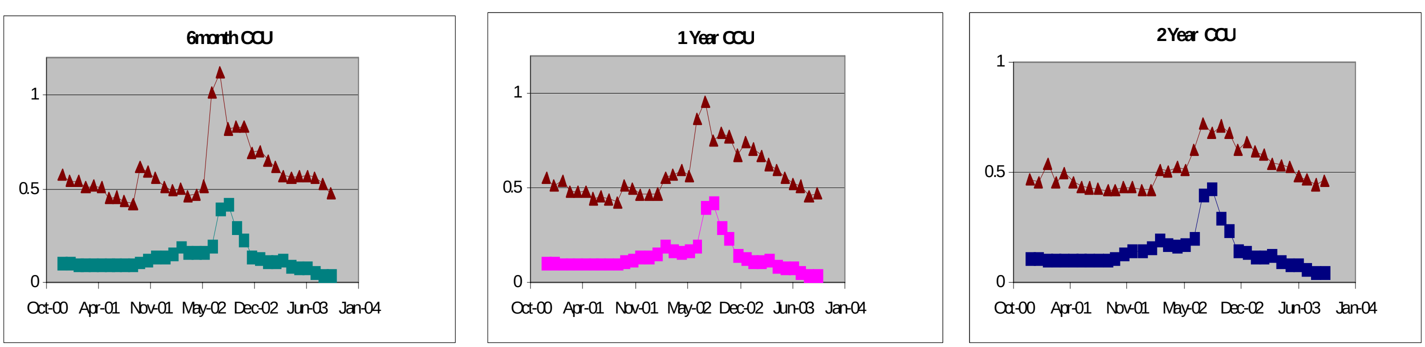
Set A: Using 0% OTM Implied Equity Volatility



Set B: Using 20% OTM Implied Equity Volatility



Set C : Using 50% OTM Implied Equity Volatility



□ Credit Spreads

△ Implied Volatility

Table 1 : Measuring the Strength between Credit Spreads and Implied Equity Volatility

Firm	Term	Moneyness	Pearson	Spearman	Confidence Interval
CCU	6M	0%	61.1%	51.0%	99%
F	6M	0%	85.9%	68.3%	99%
DPH	6M	0%	51.8%	42.2%	95%
IBM	6M	0%	64.1%	74.1%	99%
CCU	1Y	0%	69.0%	51.5%	99%
F	1Y	0%	90.0%	74.7%	99%
DPH	1Y	0%	51.3%	41.4%	95%
IBM	1Y	0%	59.6%	68.3%	99%
CCU	2Y	0%	78.2%	57.4%	99%
F	2Y	0%	90.4%	73.1%	99%
DPH	2Y	0%	51.5%	36.9%	95%
IBM	2Y	0%	55.2%	70.5%	99%
CCU	6M	10%	66.1%	56.6%	99%
F	6M	10%	86.1%	68.5%	99%
DPH	6M	10%	55.8%	45.7%	99%
IBM	6M	10%	64.3%	73.4%	99%
CCU	1Y	10%	72.0%	56.7%	99%
F	1Y	10%	91.0%	76.0%	99%
DPH	1Y	10%	54.2%	43.7%	99%
IBM	1Y	10%	59.7%	68.1%	99%
CCU	2Y	10%	77.6%	58.9%	99%
F	2Y	10%	91.6%	78.2%	99%
DPH	2Y	10%	54.2%	40.1%	95%
IBM	2Y	10%	55.8%	71.6%	99%
CCU	6M	20%	71.3%	60.6%	99%
F	6M	20%	88.5%	69.0%	99%
DPH	6M	20%	61.3%	50.4%	99%
IBM	6M	20%	63.4%	72.6%	99%
CCU	1Y	20%	75.0%	60.2%	99%
F	1Y	20%	90.0%	78.0%	99%
DPH	1Y	20%	57.4%	48.5%	99%
IBM	1Y	20%	60.1%	67.7%	99%
CCU	2Y	20%	79.6%	58.8%	99%
F	2Y	20%	92.2%	80.4%	99%
DPH	2Y	20%	57.7%	46.2%	99%
IBM	2Y	20%	56.6%	72.2%	99%
CCU	6M	30%	72.0%	54.6%	99%
F	6M	30%	86.1%	72.9%	99%
DPH	6M	30%	64.1%	55.5%	99%
IBM	6M	30%	61.4%	67.2%	99%
CCU	1Y	30%	76.0%	63.7%	99%
F	1Y	30%	91.0%	83.0%	99%
DPH	1Y	30%	58.4%	52.9%	99%
IBM	1Y	30%	60.9%	65.8%	99%
CCU	2Y	30%	79.0%	59.8%	99%
F	2Y	30%	93.4%	85.2%	99%
DPH	2Y	30%	58.9%	52.6%	99%
IBM	2Y	30%	56.4%	69.4%	99%
CCU	6M	40%	73.9%	45.3%	99%
F	6M	40%	82.6%	73.6%	99%
DPH	6M	40%	63.7%	55.9%	99%
IBM	6M	40%	56.8%	59.4%	99%
CCU	1Y	40%	76.0%	67.6%	99%
F	1Y	40%	89.0%	58.0%	99%
DPH	1Y	40%	57.5%	51.4%	99%
IBM	1Y	40%	59.7%	61.8%	99%
CCU	2Y	40%	76.3%	53.2%	99%
F	2Y	40%	92.8%	88.0%	99%
DPH	2Y	40%	58.5%	53.3%	99%
IBM	2Y	40%	55.0%	66.0%	99%
CCU	6M	50%	72.2%	41.6%	99%
F	6M	50%	79.4%	75.7%	99%
DPH	6M	50%	64.4%	59.8%	99%
IBM	6M	50%	50.9%	48.8%	99%
CCU	1Y	50%	73.0%	61.9%	99%
F	1Y	50%	85.0%	80.7%	99%
DPH	1Y	50%	60.0%	55.7%	99%
IBM	1Y	50%	50.8%	48.3%	99%
CCU	2Y	50%	72.6%	44.7%	99%
F	2Y	50%	89.9%	85.1%	99%
DPH	2Y	50%	60.9%	55.4%	99%
IBM	2Y	50%	60.1%	60.6%	99%

This table includes the Spearman Rank Order measure and the corresponding confidence Interval as well as the Pearson correlation measure for all terms across all firms. A 99% confidence interval indicates that we are 99% confident that a relationship exists between credit spreads and implied equity volatility. Both correlation measures indicate a strong positive relationship between credit spreads and implied equity volatility

Figure 2: A Time Series Graph of 5 year Implied Credit Spreads, Market Spreads and ATM Implied Equity Volatility for "F"

Implied credit spread via The Alternative Model using ATM put options with approximately 3 month maturity are calculated and compared to market quoted spreads. ATM volatility is also included in order to observe its relationship with credit spread estimates.

The correlation between the implied credit spread and market quoted spreads is 52%

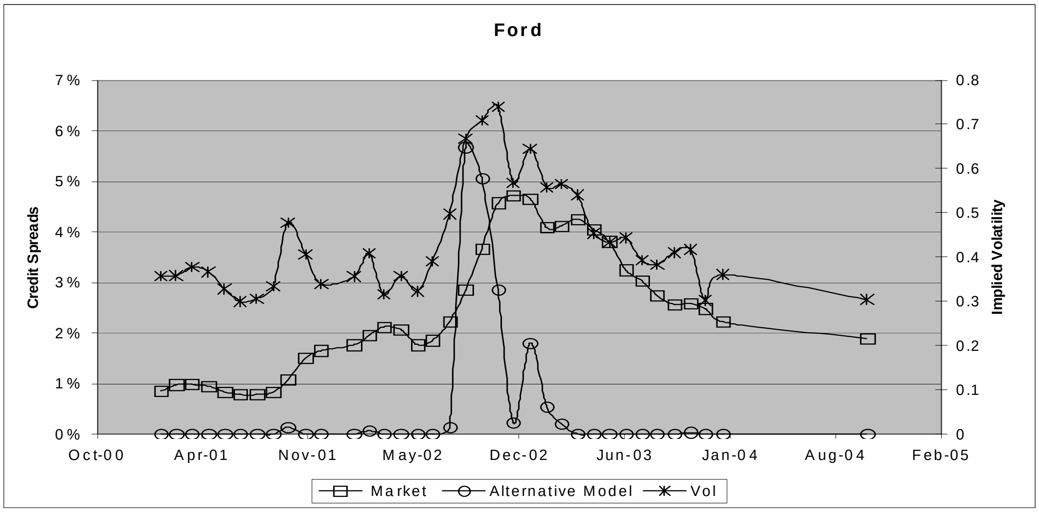


Figure 3: A Time Series Graph of 5 year Implied Credit Spreads, Market Spreads and ATM Implied Equity Volatility for "CCU"

Implied credit spread via The Alternative Model using ATM put options with approximately 3 month maturity are calculated and compared to market quoted spreads. ATM volatility is also included in order to observe its relationship with credit spread estimates.

The correlation between the implied credit spread and market spreads is 66%

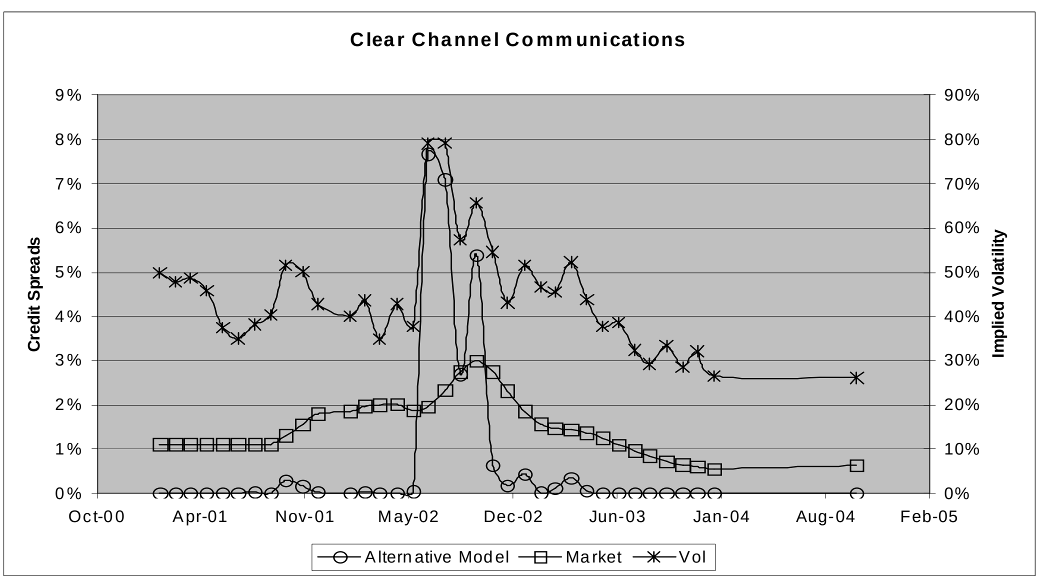
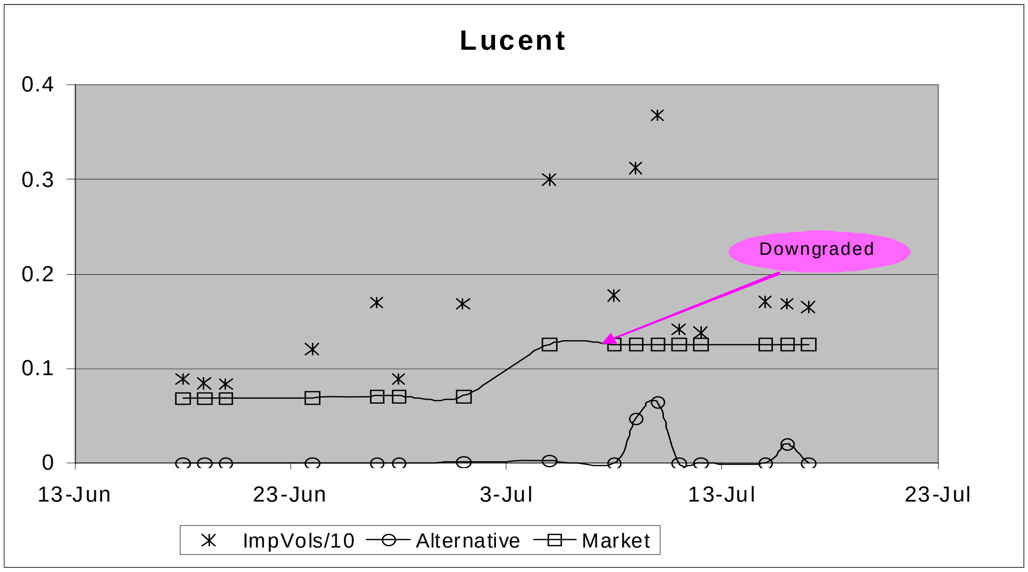


Figure 4: A Time Series Graph of 5 year Implied Credit Spreads, Market Spreads and ATM Implied Equity Volatility for "LU"

Implied credit spread via The Alternative Model using ATM put options with approximately 3 month maturity are calculated and compared to market quoted spreads. ATM volatility (which is reduced by a factor of 10) is also included.

The correlation between the implied credit spread and market quoted spreads is 42%



Graph 1: This Graph is Copied from Hull, J., I. .Nelken, and A. White "Merton's Model, Credit Risk, and Volatility Skews" Journal of Credit Risk Vol 1, No 1, pp.3-28 (2005) Figure 3

Theoretical relationship between credit spread and at-the-money volatility.
The relationship is implied by Merton's model for alternative values of the volatility skew when option maturity is two months and debt maturity is 5 years. The volatility skew is the difference between the volatility of an option with a delta of 0.25 and an option with a delta of 0.5.

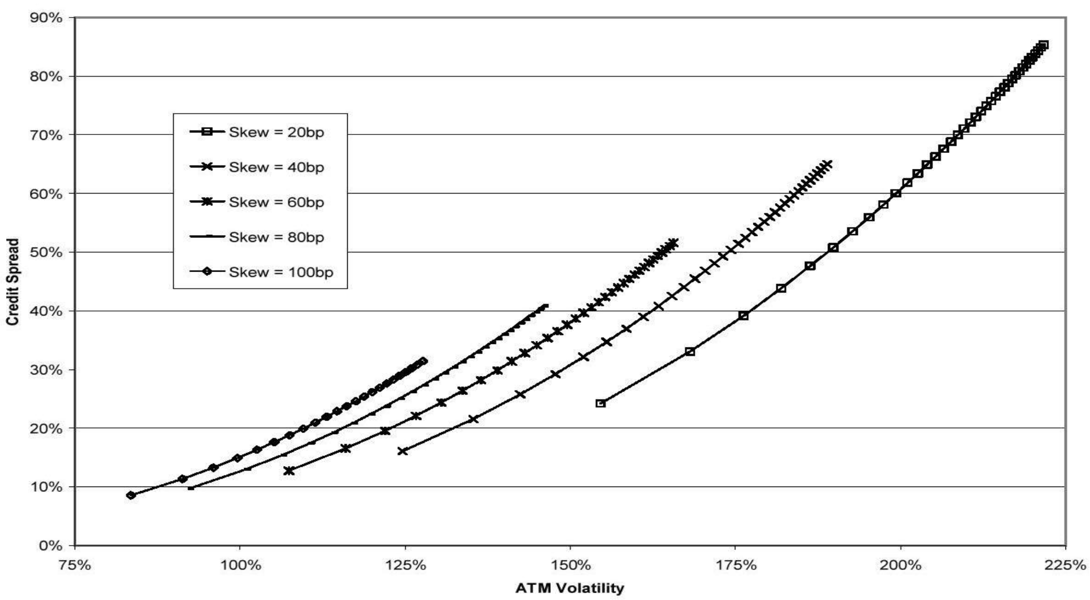


Figure 5: The Impact of Moneyness when Calculating 5 year Credit Spreads

This graph demonstrates the difference in implied credit spreads for a particular firm when ATM put options are used versus put options struck at 70% OTM., both with 3 month expiry.

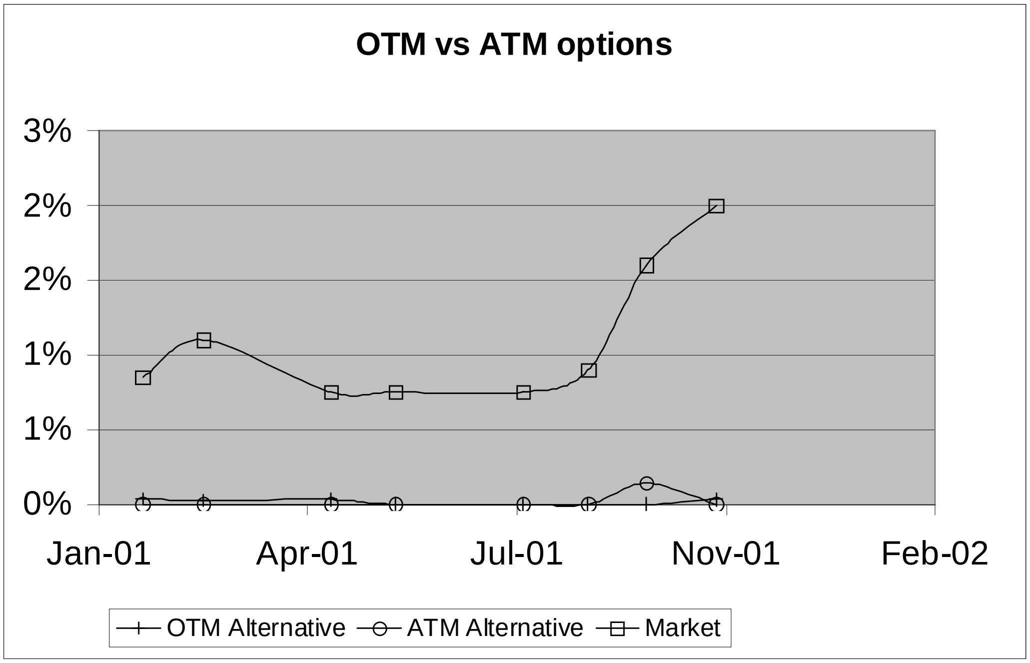


Table 2:
Implied Volatility and Stock Price for May 11, 2005

Ticker	Stock	Strike	Implied Volatilty
AA	27.54	20	21%
AET	76.24	70	27%
AGN	76.19	70	23%
ALL	57.16	50	16%
AMD	15.36	10	42%
AN	19.28	10	36%
AOC	24.59	20	38%
APA	53.61	50	32%
ASD	43.24	40	27%
ASH	65.90	60	26%
AT	56.37	50	17%
ATI	20.71	20	35%
AVP	38.91	30	27%
AVY	52.21	50	23%
AXP	52.46	50	22%
AYE	24.55	20	29%
BCR	70.57	70	17%
BEN	68.85	60	23%
BHI	42.46	40	32%
BJS	47.76	40	33%
BK	27.90	20	25%
BLI	11.28	10	48%
BLS	26.07	20	19%
BMS	27.01	20	27%
BNI	49.43	40	20%
BOL	77.41	70	21%
BSC	94.22	90	26%
CB	82.52	80	19%
CBE	66.33	60	40%
CC	15.96	10	36%
CCL	49.30	40	28%
CCU	29.98	20	35%
CEG	53.07	50	21%
CFC	34.25	30	40%
CI	96.51	90	26%
CIN	41.12	40	17%
CLX	57.88	50	20%
CMA	56.16	50	16%
COL	46.75	40	22%
CTL	30.39	30	22%
CTX	58.12	50	39%
D	70.42	70	19%
DCN	11.66	10	44%
DD	46.64	40	21%
DIS	26.80	20	24%
DLX	39.74	30	30%
DRI	30.91	30	37%
DTE	46.17	40	15%
DUK	27.68	20	24%
DVN	42.76	40	35%
ECL	31.77	30	21%
EFX	34.51	30	26%
EIX	36.92	30	21%
EK	26.35	20	28%
EMC	13.38	10	44%
EOG	45.27	40	35%
EOP	32.09	30	19%
EQR	35.17	30	18%
ETN	58.51	50	21%
EXC	47.16	40	18%
FDX	85.75	80	22%
FE	44.18	40	27%
FLR	56.77	50	27%
FRE	61.87	60	27%
GAS	37.54	30	23%
GCI	75.99	70	17%
GDT	73.29	70	18%
GIS	49.47	40	20%
GP	34.00	30	34%
GS	101.71	100	26%
GT	13.11	10	54%
GWW	54.84	50	24%
HAL	41.00	40	31%
HLT	21.94	20	28%
HRB	49.92	40	29%
IBM	72.70	70	22%
IGT	27.40	20	29%
IP	33.10	30	27%
IPG	12.58	10	43%
ITT	91.75	90	28%
JCI	54.63	50	24%
JNJ	67.78	60	15%
JP	49.75	40	28%
JWN	54.66	50	31%
KBH	57.31	50	38%
KMB	62.45	60	16%
KMG	76.60	70	44%
KR	16.20	10	29%
KRI	63.67	60	19%
KSS	49.72	40	29%
LIZ	37.50	30	23%
LLY	59.41	50	25%
LNC	42.89	40	19%
LOW	52.75	50	21%
LXK	65.32	60	29%
MAR	61.35	60	25%
MAS	30.20	30	29%
MAT	18.07	10	34%
MER	53.92	50	28%
MHS	52.11	50	29%
MIL	50.05	50	27%
MMC	28.63	20	28%
MMM	76.09	70	19%
MON	61.42	60	27%
MOT	15.77	10	29%
MRK	33.61	30	28%
MRO	45.83	40	28%
MU	10.11	10	40%
MWD	49.75	40	33%
MWV	29.93	20	42%
MYG	10.52	10	75%
NAV	28.89	20	39%
NCC	33.96	30	23%
NCR	36.74	30	29%
NI	23.63	20	38%
NKE	77.05	70	23%
NUE	48.24	40	49%
NYT	32.73	30	21%
OMC	83.28	80	19%
PBI	44.75	40	24%
PD	80.93	80	38%
PEP	56.75	50	15%
PFG	39.74	30	23%
PG	55.76	50	15%
PGL	40.07	40	16%
PGN	42.97	40	20%
PHM	71.58	70	39%
PLD	40.12	40	17%
PNW	42.66	40	16%
PPL	55.85	50	12%
PX	45.21	40	16%
RBK	41.61	40	22%
RDC	25.58	20	40%
ROH	43.49	40	32%
ROK	48.01	40	31%
RSH	25.26	20	32%
SBC	23.27	20	29%
SBL	11.07	10	48%
SDS	33.71	30	34%
SEE	48.57	40	25%
SLE	20.78	20	21%
SLM	48.17	40	25%
SPG	68.20	60	20%
SRE	38.32	30	22%
STI	72.48	70	16%
STJ	38.30	30	29%
SVU	31.33	30	22%
T	18.68	10	19%
TER	11.76	10	53%
TGT	49.10	40	24%
TUP	21.63	20	27%
TXN	26.40	20	28%
TXU	81.27	80	29%
TYC	28.68	20	29%
UCL	54.85	50	28%
UPS	72.29	70	21%
UST	43.82	40	28%
VZ	34.33	30	24%
WAG	44.16	40	27%
WB	51.59	50	19%
WFC	59.92	50	19%
WM	41.38	40	20%
WMI	29.05	20	25%
WY	64.55	60	26%
WYE	44.81	40	27%
X	38.92	30	54%
XL	74.55	70	20%
XOM	55.31	50	23%
XRX	13.17	10	49%
YUM	48.43	40	27%

Ninety liquid ATM US exchange traded put options are examined for a particular day.

The average implied volatility is 27% with standard deviation of 9%.

Figure 6: Time Series Graph of 5 yr Calculated Credit and Market Spreads for "F" using a Constant Barrier

Credit Spreads are calculated using a first-passage approach with deterministic barrier equal to the Debt Face value.

Correlation is calculated to be 54%

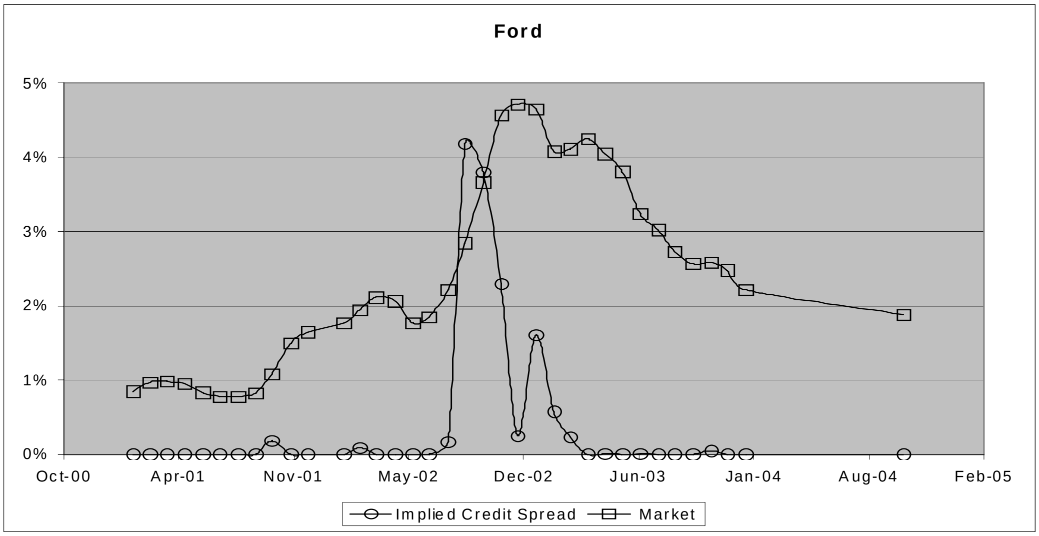


Figure 7: Time Series Graph of 5 yr Calculated Credit and Market Spreads for Ford using a Deterministic Barrier

Credit Spreads are calculated using a first-passage approach with deterministic barrier equal to the Debt Face value discounted to today.

Correlation is calculated to be 56%

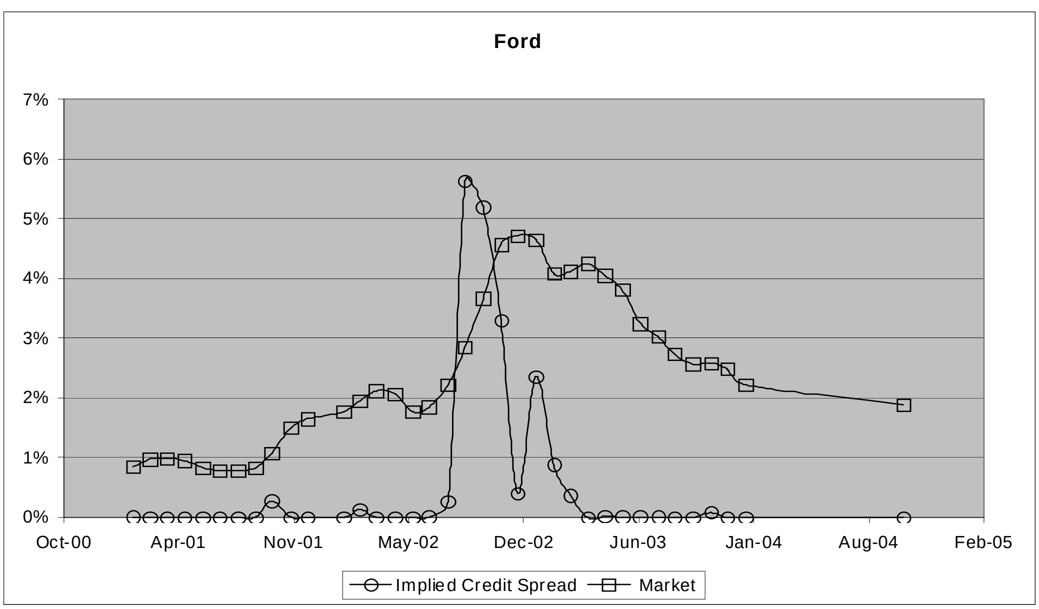


Figure 8: Time Series Graph of 5 yr Calculated Credit Spreads, Market Spreads and Implied Equity Volatility for "F" using a Random Barrier

Credit Spreads are calculated using a first-passage approach using a random barrier with uniform distribution.

Correlation is calculated to be 87%

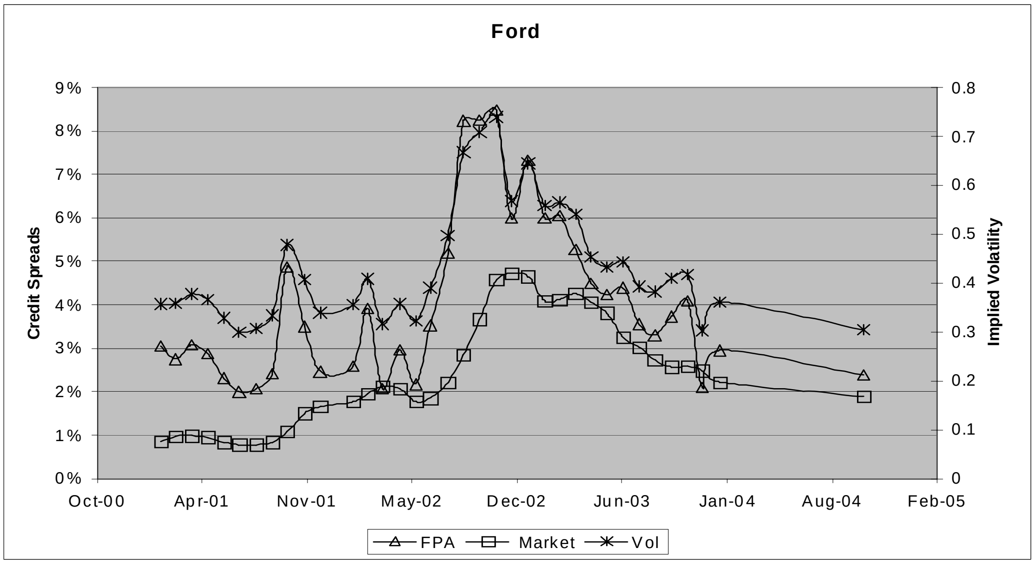


Figure 9: Time Series Graph of 5 yr Calculated Credit Spreads, Market Spreads and Implied Equity Volatility for "CCU" using a Random Barrier

Credit Spreads are calculated using a first-passage approach using a random barrier with uniform distribution.

Correlation is calculated to be 83%

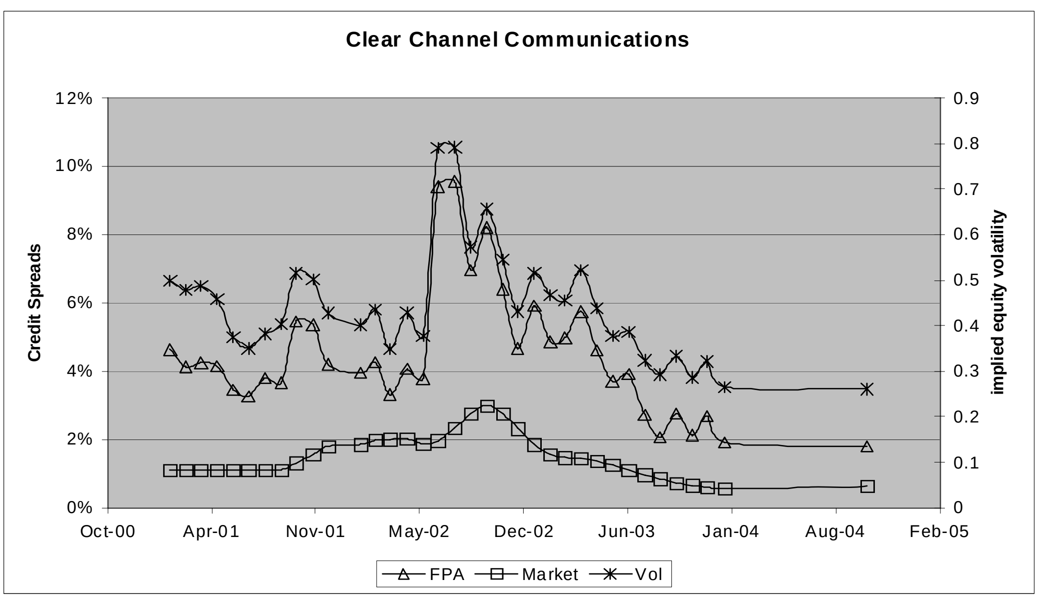


Figure 10: Time Series Graph of 5 yr Calculated Credit Spreads, Market Spreads and Implied Equity Volatility for "LU" using a Random Barrier commencing June 18, 2002 to July 18, 2002.

Credit Spreads are calculated using a first-passage approach using a random barrier with uniform distribution.

Correlation is calculated to be 67%

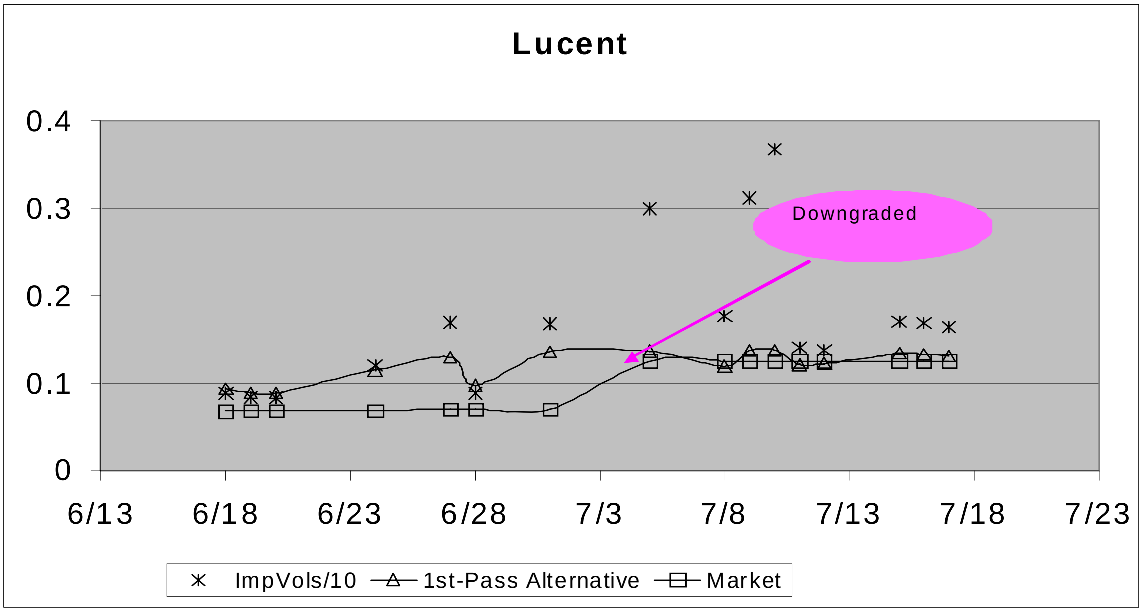


Figure 11: Theoretical relationship between credit spread and At-The-Money volatility

The relationship is implied by the FPA Model for alternative values of the volatility skew when option maturity is two months and debt maturity is 5 years.

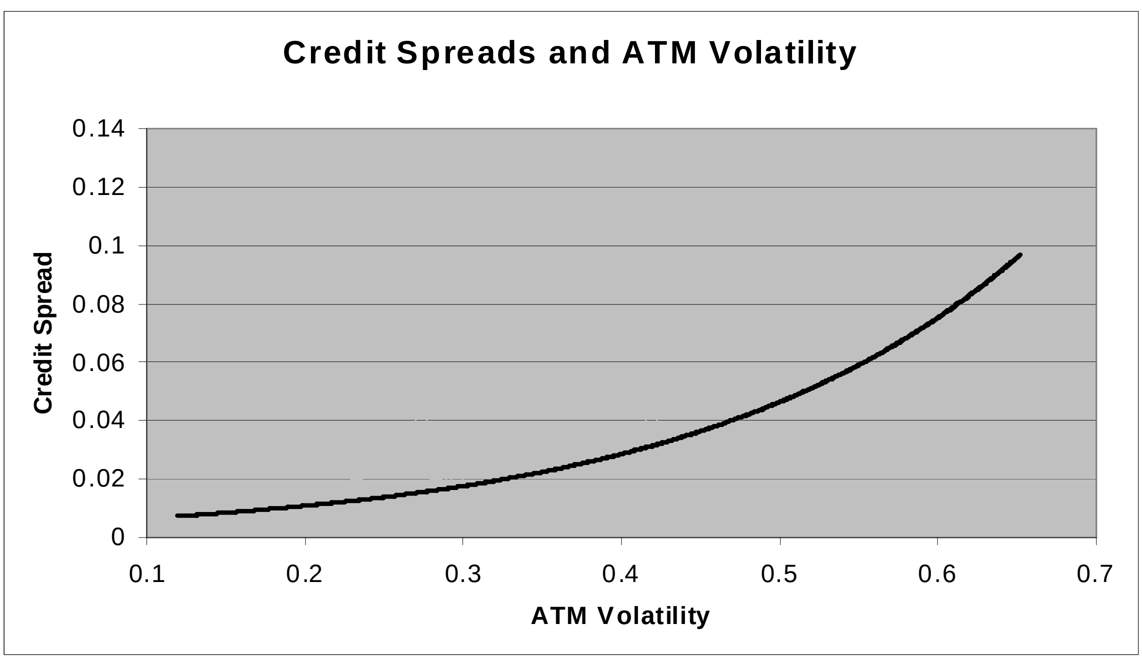


Table 3:
A Comparison of Correlations Between the Alternative Model and The FPA Model

Firm	Correlation for The Alternative Model	Correlation for FPA Model	Alternative Model Median	FPA Model Median	Market Median
AA	21.44%	40.62%	0.00%	3.68%	0.35%
ABK	-11.18%	31.21%	0.00%	3.45%	0.37%
ABS	22.68%	61.38%	0.00%	3.14%	0.75%
ADI	43.63%	52.61%	1.01%	7.14%	0.78%
AEP	58.82%	85.68%	0.00%	2.04%	1.25%
AHC	-6.43%	22.69%	0.00%	2.66%	0.82%
AIG	53.27%	72.90%	0.00%	2.62%	0.32%
AOC	68.77%	86.29%	0.00%	2.73%	0.65%
APD	14.47%	54.67%	0.00%	2.33%	0.30%
AWE	80.75%	94.25%	0.06%	5.32%	1.96%
BAX	47.68%	57.32%	0.00%	2.60%	0.51%
BHI	4.18%	28.10%	0.01%	4.37%	0.45%
BJS	37.59%	64.11%	0.00%	5.23%	1.00%
BSC	34.43%	63.48%	0.00%	3.14%	0.65%
CA	86.64%	89.51%	1.60%	7.48%	2.28%
CAG	-9.79%	24.59%	0.00%	1.79%	0.60%
CAT	41.21%	62.40%	0.00%	3.04%	0.42%
CB	38.71%	75.43%	0.00%	2.66%	0.45%
CCE	-9.02%	38.96%	0.00%	3.38%	0.40%
CD	42.65%	47.26%	0.47%	5.41%	2.73%
CIN	14.08%	29.50%	0.00%	1.57%	1.10%
CL	13.53%	61.48%	0.00%	1.90%	0.16%
CLX	18.48%	59.50%	0.00%	2.64%	0.28%
COF	70.52%	81.62%	0.13%	5.17%	2.19%
CSC	57.15%	74.91%	0.05%	4.74%	0.75%
D	19.90%	76.18%	0.00%	1.78%	0.80%
DD	36.11%	46.18%	0.00%	2.59%	0.33%
DE	10.66%	27.23%	0.00%	2.94%	0.51%
DELL	67.00%	91.20%	0.11%	4.73%	0.38%
DPH	42.72%	60.05%	0.00%	3.90%	1.45%
DTE	44.58%	59.77%	0.00%	1.73%	0.90%
DVN	51.38%	57.91%	0.01%	3.81%	0.95%
EC	65.76%	73.36%	0.00%	2.32%	0.35%
EIX	22.85%	39.83%	0.02%	5.33%	3.68%
EMR	35.40%	41.60%	0.00%	2.85%	0.48%
F	51.51%	86.63%	0.00%	3.51%	2.15%
FD	49.61%	51.91%	0.00%	3.44%	0.87%
FDX	11.66%	36.29%	0.00%	2.85%	0.55%
GM	48.25%	61.42%	0.00%	3.42%	2.65%
GPS	66.73%	79.30%	0.76%	6.34%	2.45%
GS	19.97%	65.20%	0.03%	4.10%	0.53%
HD	18.13%	29.46%	0.00%	3.66%	0.34%
HNZ	45.81%	81.84%	0.00%	1.65%	0.35%
HPQ	33.54%	77.37%	0.00%	4.12%	0.43%
IBM	48.21%	66.99%	0.00%	3.49%	0.42%
IP	60.76%	69.03%	0.00%	2.57%	0.83%
IR	13.23%	36.26%	0.00%	2.84%	0.45%
JNY	12.46%	27.34%	0.00%	3.46%	0.80%
JWN	19.46%	40.93%	0.00%	3.71%	0.85%
K	27.52%	35.16%	0.00%	1.70%	0.40%
KMB	9.21%	37.84%	0.00%	2.03%	0.16%
KMG	-7.03%	13.46%	0.00%	2.40%	0.73%
KMI	51.58%	59.21%	0.00%	2.90%	0.85%
KSE	22.47%	43.40%	0.00%	1.67%	0.50%
LMT	13.66%	44.76%	0.00%	3.08%	0.70%
LOW	41.43%	61.81%	0.00%	3.29%	0.40%
LU	48.02%	54.86%	1.01%	8.09%	6.82%
MAS	5.19%	50.13%	0.00%	2.56%	0.87%
MAY	27.81%	88.06%	0.00%	3.21%	0.51%
MCK	-5.11%	49.15%	0.00%	3.41%	1.20%
MER	61.74%	88.21%	0.02%	4.30%	0.60%
MO	22.64%	36.21%	0.00%	2.99%	1.00%
MOT	58.41%	66.96%	0.30%	6.46%	2.22%
NI	70.05%	72.00%	0.00%	3.15%	1.70%
NSC	6.22%	24.98%	0.00%	3.40%	0.54%
ORCL	69.46%	79.72%	0.21%	5.96%	0.40%
R	-5.14%	36.92%	0.00%	3.20%	0.85%
ROH	34.33%	67.39%	0.00%	3.01%	0.45%
ROK	31.98%	62.85%	0.00%	3.22%	0.80%
RTN	34.48%	35.53%	0.01%	4.04%	1.30%
SLR	41.69%	64.18%	1.34%	8.00%	6.04%
SPG	54.65%	74.55%	0.00%	1.15%	0.57%
SUNW	29.75%	58.62%	0.72%	7.08%	0.95%
SVU	20.36%	29.91%	0.00%	2.81%	1.78%
SWY	30.59%	81.48%	0.00%	3.60%	0.77%
SYY	-6.37%	10.79%	0.00%	2.03%	0.32%
T	-5.42%	37.46%	0.00%	4.01%	1.95%
TIN	29.87%	48.40%	0.00%	2.77%	1.55%
TXU	86.53%	92.39%	0.00%	2.80%	1.85%
UNH	11.29%	54.38%	1.47%	6.62%	0.40%
UST	15.85%	49.59%	0.00%	2.10%	0.60%
UTX	25.32%	57.24%	0.00%	2.81%	0.30%
VC	25.71%	57.59%	0.00%	4.40%	2.20%
VFC	-1.97%	35.09%	0.00%	2.63%	0.37%
VZ	52.67%	74.34%	0.00%	3.03%	0.86%
WHR	9.23%	24.87%	0.00%	3.41%	0.60%
WMT	-11.42%	56.61%	0.00%	2.88%	0.21%
WY	43.44%	63.28%	0.00%	3.14%	0.85%
WYE	41.42%	78.56%	0.00%	3.11%	0.55%
ZION	36.84%	67.39%	0.00%	2.00%	1.85%

Monthly credit spreads are produced using The Alternative and FPA Model for the last 4 years. The correlation between in spreads and market spreads for each times series are indicated in the table.

Median for market quotes and calculated spreads are included in order to shed light on the magnitude implied by both models.

Appendix:

System of Equations to be solved under a First-Passage-Methodology

First we must derive a Put-on-a-DOC; to the best of our knowledge, there is no published equation therefore we develop one. The following derivations in its entirety can be found in Elkhodiry (2007), however for the purpose of illustration we provide the general framework below.

The Derivation

Assume a put today with maturity *t* and strike K_1, $Put(A_0, K_1, t)$ on a DOC with strike K_2 and maturity *T* which starts at *t*, (ie. time-to-maturity of *T-t*), $DOC(A_t, K_2, T-t)$. If at time = *t,* the value of the DOC is less than K_1, the option is exercised. Therefore we can assume some critical asset value $A*$ exists such that $DOC(A*, K_2, T-t) = K_1$ and if $A_t < A*$ exercise occurs.

Therefore we value a put-on-a-DOC option today as follows:

$$Put_{DOC} = e^{-rt} E_0^Q [K_1 - DOC(A_t, K_2, T-t) 1_{(A_t < A*)}\ ] \tag{A.1}$$

We use Merton's derivation of a DOC (see Merton (1973) for details) where we consider a deterministic barrier (B) which is equal to some fraction of the discounted debt:

$$DOC = [AN(h_1) - De^{-r\tau} N(h_2)] - [BN(h_3) - (\frac{A}{B}) De^{-r\tau} N(h_4)] \tag{A.2}$$

where:

$$h_1 = [\ln(A/D) + (r + \frac{1}{2}\sigma^2)\tau] / \sqrt{\sigma^2 \tau},$$

$$h_2 = [\ln(A/D) + (r - \frac{1}{2}\sigma^2)\tau] / \sqrt{\sigma^2 \tau},$$

$$h_3 = [2\ln(B/D) - \ln(A/D) + (r + \frac{1}{2}\sigma^2)\tau] / \sqrt{\sigma^2 \tau},$$

$$h_4 = [2\ln(B/D) - \ln(A/D) + (r - \frac{1}{2}\sigma^2)\tau] / \sqrt{\sigma^2 \tau}.$$

$$B = bDe^{-r\tau} \quad for\ 0 \le b \le 1 \ , \quad \tau = T - t,$$

Therefore:

$$Put_{DOC} = e^{-rt} E_0^Q [[K_1 - (call - (BN(h_3) - (\frac{A_t}{B}) De^{-r\tau} N(h_4))] 1_{(A_t < A^*)}]$$

$$= \underbrace{e^{-rt} E_0^Q [K_1 - call 1_{(A_t < A^*)}]}_{\text{Part 1}} + \underbrace{e^{-rt} E_0^Q [BN(h_3) 1_{(A_t < A^*)}]}_{\text{Part 2}} - \underbrace{e^{-rt} E_0^Q [(\frac{A_t}{B}) De^{-r\tau} N(h_4) 1_{(A_t < A^*)}]}_{\text{Part 3}} \quad \text{(A.3)}$$

where: call = c(A,D, τ *) is the Black-Scholes call option equation.*

Since Part 1 (Put-on-Call) is already known, derivations only for Parts 2 and 3 are needed. For illustration we highlight the derivation for Part 2.

$$\text{Part 2} = e^{-rt} E_0^Q [BN(h_3) 1_{(A_t < A^*)}] \quad \text{(A.4)}$$

We let $A_t = A_0 e^u$ and $w = (r - \frac{1}{2}\sigma^2)$ where $u \sim N(wt, \sigma^2 t)$ such that:

$$\text{Part 2} = e^{-rt} B \int_{-\infty}^{\ln(A^*/A_0)} [\int_{-\infty}^{h_3} \frac{e^{-\frac{1}{2}c^2}}{\sqrt{2\pi}} dc] \frac{e^{-\frac{1}{2}(\frac{u-wt}{\sigma\sqrt{t}})^2}}{\sqrt{2\pi t \sigma^2}} du$$

$$= e^{-rt} B \int_{-\infty}^{\ln(A^*/A_0)} \int_{-\infty}^{h_3} \frac{e^{-\frac{1}{2}\{(\frac{u-wt}{\sigma\sqrt{t}})^2 + c^2\}}}{2\pi\sigma\sqrt{t}} dcdu \quad \text{(A.5)}$$

To solve this integral we use the solution of the standard bivariate normal density $M(X_1, X_2; \rho)$ function given by:

$$M(X_1, X_2; \rho) = \int_{-\infty}^{X_1} \int_{-\infty}^{X_2} \frac{1}{2\pi\sqrt{1-\rho^2}} \exp\{-\frac{1}{2}(\frac{X_1^2 - 2\rho X_1 X_2 + X_2^2}{1-\rho^2})\} dX_1 dX_2 \quad \text{(A.6)}$$

We let $u = a_0 x_1 + b_0$ and $c = a_1 x_1 + b_1 x_2$, substitute into equation (A.5) and change the limits of integration. The same logic is applied to Part 3.

Furthermore it can be shown (see Elkhodiry 2007), that our equity expansion of equation (5) becomes:

$$E_0 = A_0[N(d_1) - LN(d_2) - bLN(h_3^0) + \frac{1}{b}N(h_4^0)] \tag{A.9}$$

where:

$$h_3^0 = [2\ln(B/D) - \ln(A_0/D) + (r + \frac{1}{2}\sigma^2)T]/\sqrt{\sigma^2 T},$$

$$h_4^0 = [2\ln(B/D) - \ln(A_0/D) + (r - \frac{1}{2}\sigma^2)T]/\sqrt{\sigma^2 T}.$$

And our expansion of equation (10) results in:

$$\kappa = \frac{\alpha N(d_{1,t}) - LN(d_{2,t}) - bLN(h_3^*) + (\frac{\alpha}{b})N(h_4^*)}{N(d_1) - LN(d_2) - bLN(h_3^0) + (\frac{1}{b})N(h_4^0)} \tag{A.10}$$

where:

$$h_3^* = [2\ln(B/D) - \ln(A_t^*/D) + (r + \frac{1}{2}\sigma^2)\tau]/\sqrt{\sigma^2 \tau},$$

$$h_4^* = [2\ln(B/D) - \ln(A_t^*/D) + (r - \frac{1}{2}\sigma^2)\tau]/\sqrt{\sigma^2 \tau}.$$

$A_\tau^* = \alpha A_0 e^{r\tau}$ and $K = \kappa E_0 e^{r\tau}$

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