ARTICLE IN PRESS

ECMODE-01701; No of Pages 8

Economic Modelling xxx (2009) xxx–xxx

Contents lists available at ScienceDirect

Economic Modelling

journal homepage: www.elsevier.com/locate/econbase

Single and Double Black–Cox: Two approaches for modelling debt restructuring ☆

Isabel Abínzano [a], Luis Seco [b], Marcos Escobar [c,*], Pablo Olivares [c]

[a] Department of Business Administration, Edificio Los Madroños, Universidad Pública de Navarra, 31006 Pamplona, Navarra, Spain
[b] Department of Mathematics, University of Toronto, 100 St. George St., Toronto, ON, Canada M5S3G3
[c] Department of Mathematics, Ryerson University, 245 Church Street, Toronto, ON, Canada M5B 2K3

ARTICLE INFO

Article history:
Accepted 24 February 2009
Available online xxxx

JEL classification:
G12
G32
G33

Keywords:
First-passage-time model
Reorganization
Liquidation
Debt restructuring

ABSTRACT

In this paper we propose two first-passage-time approaches for pricing debt and equity when the firm is able to restructure its debt as an alternative to liquidation. In contrast to other first passage models that account for reorganization, our approaches allow the firm to restructure its debt by changing its maturity and/or its face value. The first approach developed consists of a first-passage model for reorganization together with a Merton approach for default, while the second approach uses first-passage models for both reorganization and default. We also provide a comparison of the proposed approaches with the Merton (Merton, R.C., 1974. On the pricing of corporate debt: The risk structure of interest rate. Journal of Finance, 29, 449–470.) and Black and Cox (Black, F. and Cox, J.C., 1976, Valuing corporate securities: Some effects of bond indenture provisions. Journal of Finance, 31, 351–367.) models.

1. Introduction

Corporate bankruptcy is very common. Indeed, hundreds of thousands of firms around the world declare bankruptcy each year. According to Beck et al. (2006), a company experiences financial distress when it encounters liquidity problems in meeting its financial liabilities.

When a corporation becomes insolvent and bankruptcy proceedings commence, the corporation may take two routes: liquidation or reorganization. In liquidation, the assets of the corporation are sold, either piecemeal or as a going concern. The proceeds from this sale are then divided among those with rights against the corporation, according to their ranking. Reorganization, on the other hand, consists of a series of agreements between the company and its creditors allowing the company to repay its debts and alter its structure to prevent the same problems from arising again.

In order to reorganize, firms can use either an out-of-court workout or a formal Chapter 11 bankruptcy.[1] Chapter 11 provides several benefits for distressed firms, allowing them to continue operations without the "harassment" of creditors (see Gertner and Scharfstein, 1991). The main purpose of Chapter 11 is to preserve the company as an operating concern while a reorganization plan is worked out among creditors. However, the primary disadvantage of a Chapter 11 reorganization is its relative cost. Gilson et al. (1990) and Jensen (1989) suggest that, due to the high restructuring costs associated with Chapter 11, firms have an incentive to reorganize voluntarily out of court via a workout. However, workouts are thwarted by holdout problems, as shown by Chatterjee et al. (1995).

Gilson et al. (1990) investigate the incentives of financially distressed firms to restructure their debt privately rather than through formal bankruptcy. They find that financial distress is more likely to be resolved through private renegotiation when more of the firm's assets are intangible, and relatively more debt is owed to banks, and that private renegotiation is less likely to succeed when there are more distinct classes of debt outstanding. In addition, Chatterjee et al. (1996) obtain that the reorganization decision depends on the firm's degree of leverage, the severity of its liquidity crisis, the degree of coordination among its creditors and the magnitude of its economic distress. Specifically, they obtain that economically distressed firms file for Chapter 11, while economically viable firms prefer workouts.

Irrespective of the debt-restructuring procedure, the rationale commonly offered is that reorganization may enable the participants to capture a greater value than they would from a liquidation. According to Bebchuk (1988), reorganization is thought to be particularly valuable when the company's assets are worth much more if it is kept as a going concern than if it is sold piecemeal, and when there are few or even no outside buyers with both accurate information about the company and

☆ We are grateful for valuable comments from Thomas Hartman, Manuel Moreno and seminar participants at the XV Foro de Finanzas, the XXXII Simposio de Análisis Económico, the XLII Conference of the Euro Working Group in Financial Modelling and the XI Encuentro de Economía Aplicada. The first author gratefully acknowledges financial support from the Spanish Ministry of Education and Science (MEC, SEJ2006-14809-C03-01) and from a grant from the Fundación ICO, and thanks the University of Toronto, where part of this work was done, for its hospitality. The usual caveat applies.

* Corresponding author.
E-mail addresses: isabel.abinzano@unavarra.es (I. Abínzano), seco@math.toronto.edu (L. Seco), escobar@ryerson.ca (M. Escobar).

[1] Hereafter we will use Chapter 11 of the U.S. Bankruptcy Code to refer to a formal reorganization procedure, although we might use other formal procedures, such as the one introduced by the Canadian Bankruptcy and Insolvency Act (see Fisher and Martel, 2003).

0264-9993/$ – see front matter © 2009 Elsevier B.V. All rights reserved.
doi:10.1016/j.econmod.2009.02.013

sufficient resources to acquire it. In such situations, the participants will have more value to share if they retain the enterprise and divide it among themselves. On the other hand, Brown (1989) argues that when a firm is in financial distress, reorganization is often desirable because it prevents losses from either liquidation or suboptimal incentive alignment of the existing contracts.

A company can carry out a reorganization in two ways: either through changes in its operational structure, such as the sale of part of its assets or a reduction in the workforce, or by restructuring its debt. Debt restructuring consists of a modification of current characteristics of debt, by changing its maturity and/or its face value.

The literature includes several works that consider the possibility of a debt restructuring in the pricing of corporate securities. Hence, Galai et al. (2007) claim that modeling of default and reorganization is instrumental in determining the value of corporate securities and the firm's financing decisions since it affects firm value and how it is shared among claimholders. This way, Franks and Torous (1989) provide a framework where the value of equity at maturity can be seen as an American call option on the firm's assets value when the company has the possibility of restructure its debt. Longstaff (1990) points out that many types of corporate reorganizations—such as Chapter 11 bankruptcy—can be viewed as the exercise of an extension privilege and suggests the application of the extendible option analysis to the pricing of a firm's capital structure. In addition, Longstaff (1990) applies the concept of flexible writer-extendible options to the pricing of the equity of a risky levered firm where bondholders have an incentive to extend the maturity date of the debt, whereas Abínzano and Navas (2008) apply the holder-extendible options defined by Longstaff (1990) to the pricing of the equity position of firms where shareholders have the possibility of filing a debt restructuring plan. Furthermore, Dumitrescu (2007) proposes a contingent valuation model for a firm financed by two bonds with different maturities and different seniorage that allows to analyze the implications of debt renegotiation on bond prices.

Notwithstanding the above, all of these papers permit reorganization (or default) only at maturity, which means that the firm must wait until maturity before being able to default or to reorganize. In other words, the value of the firm is allowed to dwindle to nearly nothing without triggering a default. Black and Cox (1976) propose a more realistic model by allowing premature default. In their model, the default occurs as soon as the value of the firm's assets reaches a lower threshold. Thus, the default time is defined by the first-passage time of the firm value to some barrier.

Nevertheless, the standard Black and Cox model does not account for the possibility of reorganization as an alternative to liquidation. Recently, two first-passage-time models have been proposed to consider the possibility of carrying out a reorganization. Fan and Sundaresan (2000) provide a reorganization framework where the claimants negotiate at a certain (endogenous) threshold to accept a reduced level of debt service until the asset value returns above the threshold. In a later contribution, following Fan and Sundaresan (2000), Francois and Morellec (2004) present a contingent claims model where the decision for liquidating or reorganizing the firm depends only on the length of time that elapses after crossing the threshold.

As can be appreciated, debt restructuring in Fan and Sundaresan (2000) and Francois and Morellec (2004) consists of a reduction in debt service payments by equity holders during the reorganization period. A more realistic setting, however, would be one in which, as soon as the firm assets value falls to some prescribed lower threshold, the firm were able to change its maturity and/or its face value. For this reason, in this paper we propose a model for the pricing of debt and equity that allows the firm to reorganize à la Black–Cox, that is, by restructuring or defaulting before maturity as soon as some pre-agreed thresholds are exceeded, using a strategy consisting of an extension of the debt maturity and/or a modification of the amount due.

Concretely, we present two approaches for pricing debt and equity in case of debt restructuring. In the first approach, debt is restructured as soon as the value of the company crosses a lower threshold, and once reorganized, the firm is not allowed to default until the new maturity time, while in the second approach, the company may default at any time before the new maturity time. Furthermore, for both approaches we investigate the consequences for shareholders and debtholders of debt restructuring and the new characteristics of debt.

The rest of the paper is organized as follows. Section 2 provides the main mathematical results needed to develop the valuation framework. In Section 3 the single Black–Cox approach to reorganization is proposed. In the same section, an extension is added allowing first-passage models for both reorganization and default. Next, a comparison of these approaches is provided in Section 4. Finally, Section 5 concludes.

2. Mathematical setting

As we have already seen, the purpose of this paper is to price the claims of a company in a setting where debt restructuring is allowed as soon as the assets value falls to some lower threshold. Next, we introduce some mathematical setting needed to develop the valuation framework.

The required mathematical results relate to the joint distribution of the endpoint and the minimum of a Brownian motion with drift. The importance of this joint distribution lies in the equivalence between the company's moment of default and the minimum of the value of its assets. Therefore, several distributions related to the minimum of a lognormal process with constant drift and volatility are provided in the next two propositions.

Let us consider the following notation:

1. V_t stands for the value of a company's assets, while $X_t = \ln V_t$.
2. V_R denotes the reorganization threshold, with $X_R = \ln V_R$.
3. V_L denotes the liquidation threshold, with $X_L = \ln V_L$, $V_R > V_L$.
4. K_1 stands for the face value of debt, with $V_R \leq K_1$.
5. K_2 is the "re-assessed" debt face value, after debt has been reorganized, with $V_L \leq K_2$. Moreover, we assume that K_2 can be greater or smaller than K_1.

In deriving the next mathematical results, we assume the following conditions:

1. There is no recovery in the presence of default.
2. The market is free of arbitrage.
3. The interest rate, r, is known and constant through time.
4. The standard deviation of the return on the assets value, σ, is constant.
5. X_t is a stochastic process defined as follows:

$$X_t = \mu t + \sigma W_t, t > c, X_c = X_0 \tag{1}$$

where W_t stands for a standard Brownian motion and c is the initial date.

Moreover, we define the running minimum as:

$$Mn_t = \min_{c \leq s \leq t} X_s. \tag{2}$$

Additionally, the default time is defined as:

$$\tau_1 = \inf \{t > c : X_t = X_R\} \tag{3}$$

where

$$X_R < X_0 \tag{4}$$

This way, we reach the next proposition.

Please cite this article as: Abínzano, I., et al., Single and Double Black–Cox: Two approaches for modelling debt restructuring, Economic Modelling (2009), doi:10.1016/j.econmod.2009.02.013

Proposition 1. *Under previous assumptions, the following results hold* (*Based on* He et al., 1998):

1. *The cumulative of τ_1 is:*

$$P(\tau_1 > t) = P(Mn_t > X_R) = P(X_t > X_R, Mn_t > X_R) =$$
$$= P(X_t > X_R) - P(X_t > X_R, Mn_t < X_R)$$
$$= \Phi\left(\frac{X_0 - X_R + \mu(t-c)}{\sigma\sqrt{(t-c)}}\right) - \exp\left(\frac{2\mu(X_R - X_0)}{\sigma^2}\right)$$
$$\times\Phi\left(\frac{X_R - X_0 + \mu(t-c)}{\sigma\sqrt{(t-c)}}\right) \tag{5}$$

where $\phi(\cdot)$ is the cumulative distribution function of the standard Normal.

2. *The density of τ_1 is:*

$$f_{\tau_1}(t) = \phi\left(\frac{X_0 - X_R + \mu(t-c)}{\sigma\sqrt{(t-c)}}\right)\left(\frac{X_0 - X_R}{2\sigma(t-c)^{3/2}} - \frac{\mu}{2\sigma\sqrt{(t-c)}}\right)$$
$$-\exp\left(\frac{2\mu(X_R - X_0)}{\sigma^2}\right)\phi\left(\frac{X_R - X_0 + \mu(t-c)}{\sigma\sqrt{(t-c)}}\right) \tag{6}$$
$$\times\left(\frac{X_R - X_0}{2\sigma(t-c)^{3/2}} - \frac{\mu}{2\sigma\sqrt{(t-c)}}\right)$$

where $\phi(\cdot)$ is the density function of the standard Normal distribution.

3. *The joint density/distribution function of (X_t, τ_1), $p_{X,\tau_1}(x,t)dx$, can be obtained using Fokker–Planck equations ($x \geq X_R$):*

$$P(X_t \in \mathrm{d}x, \tau_1 > t) = p_{X,\tau_1}(x,t)\mathrm{d}x =$$
$$= \frac{\phi\left(\frac{x-\mu(t-c)}{\sigma\sqrt{(t-c)}}\right)}{\sigma\sqrt{(t-c)}}\left[1 - \exp\left(\frac{-\left(4(X_R - X_0)^2 - 4(X_R - X_0)x\right)}{2\sigma^2(t-c)}\right)\right]\mathrm{d}x \tag{7}$$

We should make some observations regarding the above results:

Remark 1.

1. If $\mu > 0$, then $P(\tau_1 > \infty) = 1 - \exp\left(\frac{2\mu(X_R - X_0)}{\sigma^2}\right)$ that is, there is a positive probability of never crossing the threshold X_R.
2. If, $\mu < 0$, then $P(X_R \leq X_t < \infty) \approx 0$. This means there is a large probability for the process to drop below X_R.

We should also point out that the assumption of constant volatility is required for the proposition to apply. Another alternative would be to relax the assumptions of zero recovery and constant interest rate, which would require the use of some computationally challenging joint distributions, which could be found in He et al. (1998).

As can be inferred from Proposition l, the variable τ_1 accounts for the time the re-organization threshold is crossed. Another important variable is the time the liquidation threshold is crossed, τ_2. Let us define:

$$\tau_2 = \inf\{t > \tau_1 : X_t = X_L\},\ X_L < X_R \tag{8}$$

Consequently, we can enunciate the following result:

Proposition 2. *The joint cumulative distribution for (τ_1, τ_2) is:*

$$P(\tau_1 < t_1, \tau_2 < t_2) = \int_0^{t_1} P(\tau_2 < t_2 | \tau_1 < s_1) f_{\tau_1}(s_1)\mathrm{d}s_1 \tag{9}$$

$$P(\tau 2 < t2 | \tau 1 = s1) = \Phi\left(\frac{X_R - X_L + \mu(t2 - s1)}{\sigma\sqrt{t2 - s1}}\right)$$
$$-\exp\left(\frac{2\mu(X_L - X_R)}{\sigma^2}\right)\Phi\left(\frac{X_L - X_R + \mu(t2 - s1)}{\sigma\sqrt{t2 - s1}}\right) \tag{10}$$

where $f_{\tau_1}(s_1)$ is the density function found in Proposition 1 *while $P(\tau_2 < t | \tau_1 = s_1)$ is the distribution function of a Brownian motion starting at $X_{s_1} = X_R$ with threshold X_L.*

Finally, we should remark that differentiating Eq. (9) with respect to t_1 and t_2 would lead to the joint density of (τ_1, τ_2), $f_{\tau_1,\tau_2}(t_1, t_2)$.

3. The valuation framework

In this section, we develop two extensions of the Black and Cox (1976) model with the aim of accounting for the possibility of a reorganization consisting in a restructuring of corporate debt. The first extension is a first-passage-time approach for reorganization together with a Merton approach for default, while the second extension uses first-passage-time models for both reorganization and default.

Let us think of a firm financed with shares of stock and one zero-coupon bond with face value K_1 and maturity T_1. Let us suppose that there is no possibility of reorganization. In this context, Merton (1974) proposes to consider the value of the equity as a call option on the value of the firm's assets with strike price equal to the face value of debt. In this approach, default by the company occurs only at maturity T_1. As a result, the payoff expected by shareholders at T_1 will be (European call):

$$\left(V_{T_1} - K_1\right)1_{\{V_{T_1} > K_1\}} = \left(V_{T_1} - K_1\right)^+ \tag{11}$$

while the payoff at maturity from the debtholders' perspective will be (bond minus put):

$$K_1 1_{\{V_{T_1} > K_1\}} + V_{T_1}1_{\{V_{T_1} \leq K_1\}} = K_1 - \left(-V_{T_1} + K_1\right)^+. \tag{12}$$

That is, if the value of the company is larger than its debt, debtholders get K_1 while shareholders get $V_{T_1} - K_1$, and in the event of default, debtholders receive what is left of the company, V_{T_1}, while shareholders get 0.

In contrast to Merton (1974), Black and Cox (1976) allow default to occur at anytime prior to maturity. If we consider a firm financed with equity and one zero-coupon bond with face value K_1 and maturity T_1, the payoff expected by shareholders at min (T_1, τ_1) will be (Merton if no default, zero otherwise):

$$\left(V_{T_1} - K_1\right)1_{\{\tau_1 > T_1\}}1_{\{V_{T_1} > K_1\}} = \left(V_{T_1} - K_1\right)^+ 1_{\{\tau_1 > T_1\}} \tag{13}$$

while from the view of debtholders, the payoff at min (T_1, τ_1) shall be (Merton if no default, V_R otherwise):

$$K_1 1_{\{V_{T_1} > K_1\}} + V_{T_1}1_{\{V_{T_1} \leq K_1\}} = \left(K_1 - \left(-V_{T_1} + K_1\right)^+\right)1_{\{\tau_1 > T_1\}}$$
$$+ V_R 1_{\{\tau_1 \leq T_1\}} \tag{14}$$

with $V_R \leq K_1$. In other words, as soon as the value of the company is lower than its debt, debtholders get K_1 while shareholders get 0.

Please cite this article as: Abínzano, I., et al., Single and Double Black–Cox: Two approaches for modelling debt restructuring, Economic Modelling (2009), doi:10.1016/j.econmod.2009.02.013

Conversely, if no default occurs, shareholders get $V_{T_1}-K_1$ while debtholders get the value of the debt, K_1.

Let us now assume that the firm has the possibility of carrying out a reorganization. Specifically, we understand the reorganization as an extension of the debt maturity and/or a modification of the amount due. In the case of a formal procedure—such as a Chapter 11 filing—the debtor is entitled to file a reorganization plan during a specified period of time. Creditors will accept this reorganization plan only if the value of debt after the reorganization is higher than the payments they would receive in the event of liquidation. When liquidation occurs, control of the firm is transferred from the equity to the debtholders. It is commonly accepted that this transfer is costly. Indeed, Warner (1977) distinguishes between two types of costs of liquidation: direct and indirect. Direct costs include lawyers' and accountants' fees, while indirect costs include lost sales and lost profits. In what follows we will assume that liquidation costs are high enough for debtholders to accept the reorganization plan. This assumption is consistent with Branch (2002), who points out that a significant part of the value of the bankrupt firm is consumed in dealing with its distress. Furthermore, Altman (1984) finds that the total (direct and indirect) liquidation cost amounts to about 15% of pre-distress firm value.

3.1. Single Black–Cox risky debt with reorganization

Let us consider the firm described above. Let us make the following assumptions:

1. A reorganization is allowed as soon as the asset's value crosses a threshold, V_R, with $\tau_1 \leq T_1$. Both the maturity time and the face value of debt are changed at this stage to T_2 and K_2, respectively.
2. Default may occur only at T_2 if, and only if, $V_{T_2} < K_2$.

From Section 2 it is also assumed that $V_R \leq K_1$, which means that reorganization makes sense if, and only if, the assets of the company are not enough to cover the debt. We must also remark that the new face value of debt, K_2, could be less than the initial face value, K_1. This is consistent with Haugen and Senbet (1978, 1988), who argue that debtholders could offer to reduce the debt claim so as to avoid bankruptcy and its associated costs.

It should be noted that, in contrast to Fan and Sundaresan (2000) and Francois and Morellec (2004), in our setting the distress thresholds, V_R and V_L, are inputs to the problem. As noted by Leland (1994) and Ericson and Reneby (1998), there are several ways to determine and justify a distress threshold. An economic approach views the distress threshold as the level of asset value that is required for the firm to retain sufficient credibility to continue operations. The contractual criteria to define the distress threshold are based on positive net worth covenants that enable bondholders to force reorganization or liquidation in the event of the firm value falling below a pre-determined threshold. Thus, we can see V_R and V_L as wake-up calls to the company that the value of its assets is too low for it to continue normal operations. An example might be to take V_R and V_L as K_1 and K_2, respectively.

Moreover the relationship between shareholders and debtholders is complementary, in the sense that the sum of their values should equal the value of the firm. This means that once the calculations for one party are complete, the others can be obtained straight-forwardly. Similarly, relationships like call-put parity for a European option could be obtained. For the sake of clarity, the viewpoints of both the shareholders and the debtholders will be described more fully in the sections that follow.

3.1.1. Shareholders' viewpoint

In the Single Black–Cox framework, the payoff condition satisfied at min (T_1, τ_1) from shareholders' (SH) point of view is:

$$\left(V_{T_1} - K_1\right)1_{\{\tau_1 > T_1\}}1_{\left\{V_{T_1} > K_1\right\}} + c(V_R, K_2, T_2 - \tau_1)1_{\{\tau_1 \leq T_1\}} \tag{15}$$

where $c(x, k, T)$ denotes the price of a European call option with current value of the underlying asset x, strike price k and time to maturity T. In other words, shareholders receive the same payoff as in the Black–Cox framework plus a call option in the event of reorganization.

Once we know the payoffs to be received by shareholders at min (T_1, τ_1), we use the same reasoning as Black and Cox (1976) to obtain an expression for the current value of equity.

Let us suppose the absence of arbitrage opportunities. With the assumption given by expression (1) then there exists a risk-neutral probability measure Q such that:

$$dV_t = rV_t dt + \sigma V_t dW_t^Q$$

where r is the risk-free interest rate, σ is the return volatility of V_t, and W_t^Q is a standard Brownian motion. As a consequence, we can value the firm's equity by discounting their expected value at min(T_1, τ_1) at the risk-free discount rate, r. Then we can ennunciate the following proposition:

Proposition 3. *Under previous assumptions, the price of a single Black–Cox equity with reorganization is:*

$$\begin{aligned} SBC_{SH}(V_0, V_R, K_1, K_2, T_1, T_2) &= e^{-rT_1}E_0\left[\left(\exp\left(X_{T_1}\right) - K_1\right)1_{\{\tau_1 > T_1\}}1_{\left\{X_{T_1} > \ln K_1\right\}}\right] \\ &\quad + E_0\left[e^{-r\tau_1}c(V_R, K_2, T_2 - \tau_1)1_{\{\tau_1 < T_1\}}\right] \\ &= e^{-rT_1}\int_{\ln K_1}^{\infty}(\exp(x) - K_1)p_{X,\tau_1}(x, T_1)dx \\ &\quad + \int_0^{T_1} e^{-rt}c(V_R, K_2, T_2 - t)f_{\tau_1}(t)dt \end{aligned} \tag{16}$$

where:

$$c(V_R, K_2, T_2 - t) = V_R\Phi(a) - K_2 e^{-r(T_2 - t)}\Phi(b) \tag{17}$$

with:

$$a = \frac{\ln\frac{V_R}{K_2} + \left(r + \frac{1}{2}\sigma^2\right)(T_2 - t)}{\sigma\sqrt{T_2 - t}} \tag{18}$$

$$b = a - \sigma\sqrt{T_2 - t} \tag{19}$$

and with $f_{\tau_1}(t)$ and $p_{X,\tau_1}(x,T_1)$ defined by Eqs. (6) and (7).

Proof. This follows by substituting properly from Proposition 1. □

Remark 2. The "price of reorganizing" for shareholders within the Black–Cox framework would be the difference between the value of SBC and BC:

$$E_0\left[e^{-r\tau_1}c(V_R, K_2, T_2 - \tau_1)1_{\{\tau_1 \leq T_1\}}\right] \tag{20}$$

3.1.2. Debtholders' viewpoint

On the other hand, the payoff condition satisfied at min (T_1, τ_1) from the point of view of debtholders (DH) is:

$$\begin{aligned} &K_1 1_{\{\tau_1 > T_1\}}1_{\left\{V_{T_1} > K_1\right\}} + V_{T_1}1_{\{\tau_1 > T_1\}}1_{\left\{V_{T_1} \leq K_1\right\}} \\ &\quad + \left(K_2 e^{-r(T_2 - \tau_1)} - p(V_R, K_2, T_2 - \tau_1)\right)1_{\{\tau_1 \leq T_1\}} \end{aligned} \tag{21}$$

where $p(V_R, K_2, T_2 - \tau_1)$ denotes the price of a European put option. This can be seen as the Black–Cox payoff for debtholders plus "$-V_R +$ *Bond* $-$ *Put*" (which comes from the standard call-put parity).

Assuming again the absence of arbitrage opportunities, we reach the following corollary:

Corollary 1. *Under previous assumptions, the price of a single Black–Cox risky debt with reorganization is*:

$$\begin{aligned} SBC_{DH}(V_0,V_R,K_1,K_2,T_1,T_2) &= e^{-rT_1}K_1E_0\left[1_{\{\tau_1>T_1\}}1_{\{X_{T_1}>\ln K_1\}}\right] \\ &+ e^{-rT_1}E_0\left[\exp\left(X_{T_1}\right)1_{\{\tau_1>T_1\}}1_{\{X_{T_1}\le \ln_{K_1}\}}\right] + K_2E_0\left[e^{-rT_2}1_{\{\tau_1<T_1\}}\right] \\ &- E_0\left[e^{-r\tau_1}p(V_R,K_2,T_2-\tau_1)1_{\{\tau_1<T_1\}}\right] = e^{-rT_1}K_1\int_{\ln K_1}^{\infty}p_{X,\tau_1} \\ &\times(x,T_1)\mathrm{d}x + e^{-rT_1}\int_{-\infty}^{\ln K_1}\exp(x)p_{X,\tau_1}(x,T_1)\mathrm{d}x + K_2e^{-rT_2}\int_0^{T_1}f_{\tau_1}(t)\mathrm{d}t \\ &- \int_0^{T_1}e^{-rt}p(V_R,K_2,T_2-t)f_{\tau_1}(t)dt \end{aligned} \tag{22}$$

where:

$$p(V_R,K_2,T_2-t) = K_2e^{-r(T_2-t)}\Phi(-b) - V_R\Phi(-a) \tag{23}$$

with:

$$a = \frac{\ln\left(\frac{V_R}{K_2}\right) + \left(r+\frac{1}{2}\sigma^2\right)(T_2-t)}{\sigma\sqrt{T_2-t}} \tag{24}$$

$$b = a - \sigma\sqrt{T_2-t} \tag{25}$$

and with $f_{\tau_1}(t)$ and p_{X,τ_1} (x, T_1) defined by Eqs. (6) and (7).

Proof. This follows by substituting properly from Proposition 1. □

Remark 3. The "price of reorganizing" for debtholders within the Black–Cox framework would be the difference between the value of SBC and BC:

$$E_0\left[e^{-r\tau_1}\left(K_2e^{-r(T_2-\tau_1)} - p(V_R,K_2,T_2-\tau_1) - V_R\right)1_{\{\tau_1\le T_1\}}\right] \tag{26}$$

3.2. Double Black–Cox risky debt with reorganization

This approach consists of a generalization of the Single Black–Cox in the sense that after reorganization the default may occur at any time before the newly set maturity time. As before, there would be two main assumptions in this case:

1. A reorganization is allowed as soon as the value of the assets crosses a threshold, V_R, with $\tau_1 \le T_1$. Both the maturity time and face value of debt are changed at this stage to T_2 and K_2, respectively.
2. Default may occur either before maturity T_2 as soon as $V_t < V_L$, with $V_L < V_R$ and $t > \tau_1$, or at maturity if $V_{T_2} < K_2$.

3.2.1. Shareholders viewpoint

Now, in the Double Black–Cox framework the payoff condition satisfied at min (T_1, τ_1) from shareholders' viewpoint is:

$$\left(V_{T_1}-K_1\right)1_{\{\tau_1>T_1\}}1_{\{V_{T_1}>K_1\}} + cFPM(V_R,K_2,V_L,T_2-\tau_1)1_{\{\tau_1<T_1\}} \tag{27}$$

where $cFPM(V_R, K_2, V_L, T)$ is a first passage model European option and $V_R = V(0)$, that is:

$$\begin{aligned} cFPM(V_R,K_2,V_L,T) &= e^{-rT}E\left[(\exp(X_T)-K_2)1_{\{\tau_2>T\}}1_{\{X_T>\ln K_2\}}\right] \\ &= e^{-rT}\int_{\ln K_2}^{\infty}(\exp(x)-K_2)p_X,\tau_2(x,T)\mathrm{d}x \end{aligned} \tag{28}$$

For the sake of clarity, notice that shareholders receive the same payoff as in Black and Cox (1976) plus a "first passage call" in the event of reorganization (or SBC payoff plus first passage call minus "SBC-call" in the event of reorganization).

Proposition 4. *Under previous assumptions, the price of a Double Black–Cox equity with reorganization is:*

$$\begin{aligned} DBC_{SH}(V_0,V_R,V_L,K_1,K_2,T_1,T_2) &= e^{-rT_1}E_0\left[\left(\exp\left(X_{T_1}\right)-K_1\right)1_{\{\tau_1>T_1\}}\right. \\ &\left.\times 1_{\{X_{T_1}>\ln K_1\}}\right] + E_0\left[e^{-r\tau_1}cFPM(V_R,K_2,V_L,T_2-\tau_1)1_{\{\tau_1<T_1\}}\right] \\ &= e^{-rT_1}\int_{\ln K_1}^{\infty}(\exp(x)-K_1)p_{X,\tau_1}(x,T_1)\mathrm{d}x + e^{-rT_2}\int_0^{T_1}\int_{\ln K_2}^{\infty} \\ &\times(\exp(x)-K_2)p_{X,\tau_2}(x,T_2-t_1)f_{\tau_1}(t_1)\mathrm{d}x\mathrm{d}t_1 \end{aligned} \tag{29}$$

Proof. This follows by substituting properly from Propositions 1 and 2.□

Remark 4. The "price of reorganizing" for shareholders within the Black–Cox framework would be the difference between the value of DBC and BC:

$$E_0\left[e^{-r\tau_1}cFPM(V_R,K_2,V_L,T_2-\tau_1)1_{\{\tau_1<T_1\}}\right]. \tag{30}$$

3.2.2. Debtholders' viewpoint

On the other hand, the payoff condition at min (T_1, τ_1) from debtholders' viewpoint is:

$$\begin{aligned} &K_11_{\{\tau_1>T_1\}}1_{\{V_{T_1}>K_1\}} + V_{T_1}1_{\{\tau_1>T_1\}}1_{\{V_{T_1}\le K_1\}} + \\ &+ E_{\tau_1}\left[\begin{array}{l} e^{-r(T_2-\tau_1)}K_21_{\{\tau_2>T_2\}}1_{\{V_{T_2}>K_2\}} \\ + e^{-r(T_2-\tau_1)}V_{T_2}1_{\{\tau_2>T_2\}}1_{\{V_{T_2}\le K_2\}} + e^{-r\tau_2}V_L1_{\{\tau_2\le T_2\}} \end{array}\right]\cdot 1_{\{\tau_1\le T_1\}} \end{aligned} \tag{31}$$

In parity with the above section, this can be presented as receiving BC payoff minus a "first passage call" in the event of reorganization.

Corollary 2. *The price of a Double Black–Cox risky debt with reorganization is*:

$$\begin{aligned} DBC_{DH}(V_0,V_R,V_L,K_1,K_2,T_1,T_2) &= e^{-rT_1}K_1E_0\left[1_{\{\tau_1>T_1\}}1_{\{X_{T_1}>\ln K_1\}}\right] \\ &+ e^{-rT_1}E_0\left[\exp\left(X_{T_1}\right)1_{\{\tau_1>T_1\}}1_{\{XT_1<\ln K_1\}}\right] + K_2e^{-rT_2}E_0 \\ &\times\left[1_{\{\tau_1<T_1\}}1_{\{\tau_2>T_2\}}1_{\{X_{T_2}>\ln K_2\}}\right] + e^{-rT_2}E_0\left[\exp\left(X_{T_2}\right)1_{\{\tau_1<T_1\}}\right. \\ &\left.\times 1_{\{\tau_2>T_2\}}1_{\{X_{T_2}<\ln K_2\}}\right] + V_LE_0\left[e^{-r\tau_2}1_{\{\tau_1<T_1\}}1_{\{\tau_2<T_2\}}\right] \\ &= e^{-rT_1}K_1\int_{\ln K_1}^{\infty}p_{X,\tau_1}(x,T_1)\mathrm{d}x + e^{-rT_1}\int_{-\infty}^{\ln K_1}\exp(x)p_{X,\tau_1}(x,T_1)\mathrm{d}x \\ &+ e^{-rT_2}K_2\int_0^{T_1}\left(\int_{\ln K_2}^{-\infty}p_{X,\tau_2}(x,T_2-t_1)\mathrm{d}x\right)f_{\tau_1}(t_1)\mathrm{d}t_1 + e^{-rT_2}\int_0^{T_1} \\ &\times\left(\int_{-\infty}^{\ln K_2}\exp(x)p_{X,\tau_2}(x,T_2-t_1)\mathrm{d}x\right)f_{\tau_1}(t_1)\mathrm{d}t_1 + V_L\int_0^{T_1} \\ &\times\left(\int_0^{T_2-t_1}e^{-rt_2}f_{\tau_2}(t_2)\mathrm{d}t_2\right)f_{\tau_1}(t_1)\mathrm{d}t_1. \end{aligned} \tag{32}$$

Table 1
Pricing of debt and equity using four approaches: Merton (1974), Black and Cox (1976) and the Single and Double Black–Cox models proposed in this paper.

	Equity	Debt
Merton(V_0, V_R, K_1, T_1)	0.3574	0.6426
Black–Cox(V_0, V_R, K_1, T_1)	0.3000	0.7000
SBC(V_0, V_R, K_1, K_2, T_1, T_2)	0.3967	0.6033
DBC(V_0, V_R, V_L, K_1, K_2, T_1, T_2)	0.3792	0.6208

The value of the volatility is $\sigma = 0.5$ and the risk-free interest rate is $r = 0$. The assets value at zero is $V_0 = 1$. The first maturity date is $T_1 = 1$ and the second maturity date, in the event of reorganization, is $T_2 = 2$. The thresholds for reorganization and for liquidation are $V_R = 0.7$ and $V_L = 0.5$, respectively. The debt amount at first maturity is $K_1 = 0.7$ and at second maturity is $K_2 = 0.7$.

Proof. This follows by substituting properly from Propositions 1 and 2.□

Remark 5. The "price of reorganizing" for debtholders within the Black–Cox framework would be the difference between the value of DBC and BC:

$$E_0\left[\left(E_{\tau_1}\left[\begin{array}{l} e^{-r(T_2-\tau_1)}K_2 1_{\{\tau_2>T_2\}}1_{\{X_{\tau_2}>\ln K_2\}}+\\ e^{-r(T_2-\tau_1)}\exp\left(X_{\tau_2}\right)1_{\{\tau_2>T_2\}}1_{\{X_{\tau_2}\le \ln K_2\}}+e^{-r\tau_2}V_L 1_{\{\tau_2\le T_2\}}\end{array}\right]-V_R\right)1_{\{\tau_1\le T_1\}}\right] \quad (33)$$

4. Numerical results

Once the expressions for pricing debt and equity in the event of a reorganization have been obtained, we provide some numerical results with the aim of clarifying the impact of variables and parameters on each of the two approaches we propose. Firstly, in Tables 1 and 2 we show two examples of debt and equity pricing using the four approaches studied: Merton (1974) (expressions (11) and (12)), Black and Cox (1976) (expressions (13) and (14)) and the Single (expressions (16)–(19) and (22)–(25)) and Double Black–Cox models (expressions (29) and (32)) proposed in this paper. In Table 1, the volatility value is $\sigma = 0.5$, the risk-free interest rate is $r = 0$ and the company value at zero is $V_0 = 1$. The first maturity date is $T_1 = 1$ and the second maturity date, in the event of reorganization, is $T_2 = 2$. The thresholds for reorganization and for liquidation are $V_R = 0.7$ (70% of the value of the company) which equals the debt amount at first maturity, that is $K_1 = 0.7$, and $V_L = 0.5$, respectively. The debt amount at second maturity in the event of reorganization is $K_2 = 0.7$, which equals K_1. In Table 2, K_2 is smaller than K_1, that is, debtholders offer a reduction in the face value of debt. It can be noticed that in both tables the shareholders value obtained using the Black and Cox (1976) model is lower than that given by Merton (1974). This result is reasonable because there is a higher probability of default in the former structure than in the latter. Moreover the value obtained by using the Double Black–Cox approach is lower than the value given by the Single Black–Cox approach. The reasoning for this result is similar, because, when default before maturity is allowed, the likelihood of default increases and so does the pressure on shareholders. For bondholders the opposite is obtained. We must also observe that the value of equity in the Single and Double Black–Cox approaches is higher than in the Black–Cox approach, that is, the possibility of reorganizing the firm increases the value of equity. This result is consistent with Leland (1994), who obtains that the possibility of debt restructuring increases the value of equity, and with Chi and Tang (2007), who show the potential gains from investing in the equity of firms under reorganization.

Secondly, we describe the impact of some meaningful parameters, like time to maturity, face value of debt and thresholds, on the credit spread of the firm. The credit spread can be defined as the difference between the yield of the firm's debt and the yield on an otherwise equivalent default-free zero bond, that is, the extra yield demanded by creditors to bear the potential losses due to a default (or reorganization) of the company. It is calculated as $\left\{\left(\frac{-\ln\left(\frac{B}{K_1}\right)}{T_1}\right)-r\right\}$ where B

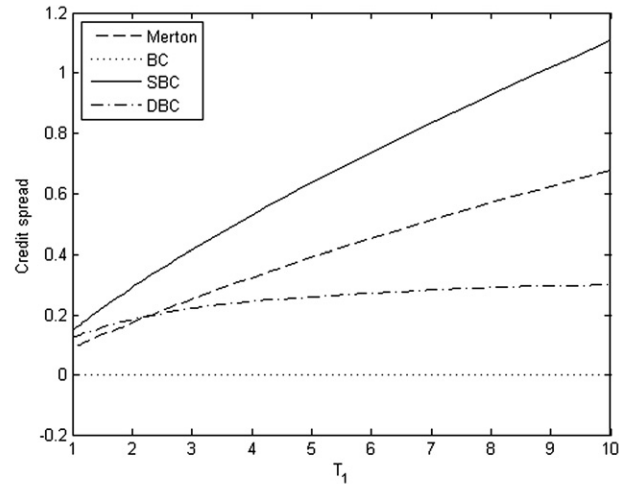

Fig. 1. Effect of maturity time on the credit spread for the following four approaches: Merton (1974), Black and Cox (1976) and the Single and Double Black–Cox models proposed in this paper. The value of the volatility is $\sigma = 0.5$ and the risk-free interest rate is $r = 0$. The assets value at zero is $V_0 = 1$. The second maturity date is $T_2 = 2T_1$. The thresholds for reorganization and for liquidation are $V_R = 0.7$ and $V_L = 0.5$, respectively. The debt amount at first maturity is $K_1 = 0.7$ and at second maturity is $K_2 = 0.7$.

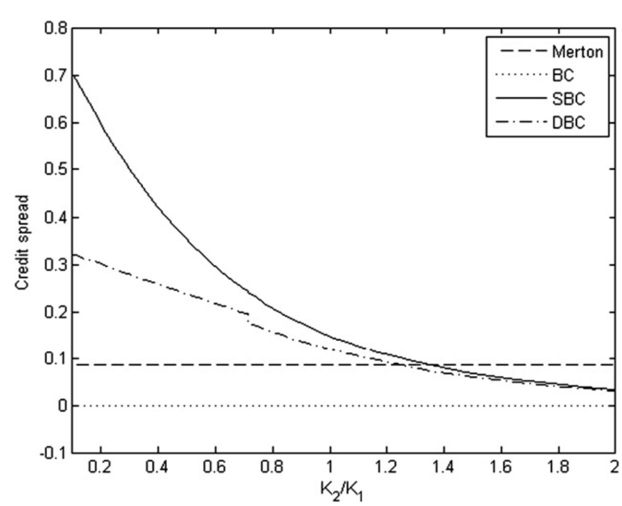

Fig. 2. Influence of the ratio K_2/K_1 on the credit spread for the following four approaches: Merton (1974), Black and Cox (1976) and the Single and Double Black–Cox models proposed in this paper. The value of the volatility is $\sigma = 0.5$ and the risk-free interest rate is $r = 0$. The assets value at zero is $V_0 = 1$. The first maturity date is $T_1 = 1$ and the second maturity date, in the event of reorganization, is $T_2 = 2$. The thresholds for reorganization and for liquidation are $V_R = 0.7$ and $V_L = 0.5$, respectively.

Table 2
Pricing of debt and equity using four approaches: Merton (1974), Black and Cox (1976) and the Single and Double Black–Cox models proposed in this paper.

	Equity	Debt
Merton(V_0, V_R, K_1, T_1)	0.3574	0.6426
Black–Cox(V_0, V_R, K_1, T_1)	0.3000	0.7000
SBC(V_0, V_R, K_1, K_2, T_1, T_2)	0.4520	0.5480
DBC(V_0, V_R, V_L, K_1, K_2, T_1, T_2)	0.4240	0.5760

The value of the volatility is $\sigma = 0.5$ and the risk-free interest rate is $r = 0$. The assets value at zero is $V_0 = 1$. The first maturity date is $T_1 = 1$ and the second maturity date, in the event of reorganization, is $T_2 = 2$. The thresholds for reorganization and for liquidation are $V_R = 0.7$ and $V_L = 0.5$, respectively. The debt amount at first maturity is $K_1 = 0.7$ and at second maturity is $K_2 = 0.5$.

Please cite this article as: Abínzano, I., et al., Single and Double Black–Cox: Two approaches for modelling debt restructuring, Economic Modelling (2009), doi:10.1016/j.econmod.2009.02.013

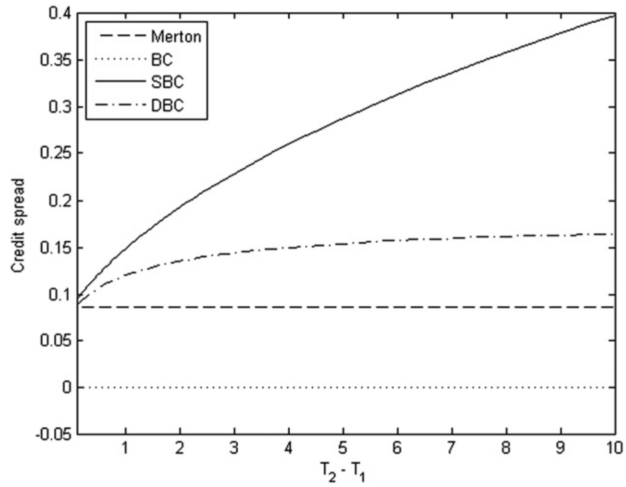

Fig. 3. Influence of T_2-T_1 on the credit spread for the following four approaches: Merton (1974), Black and Cox (1976) and the Single and Double Black–Cox models proposed in this paper. The value of the volatility is $\sigma=0.5$ and the risk-free interest rate is $r=0$. The assets value at zero is $V_0=1$. The first maturity date is $T_1=1$. The thresholds for reorganization and for liquidation are $V_R=0.7$ and $V_L=0.5$, respectively. The debt amount at first maturity is $K_1=0.7$ and at second maturity is $K_2=0.7$.

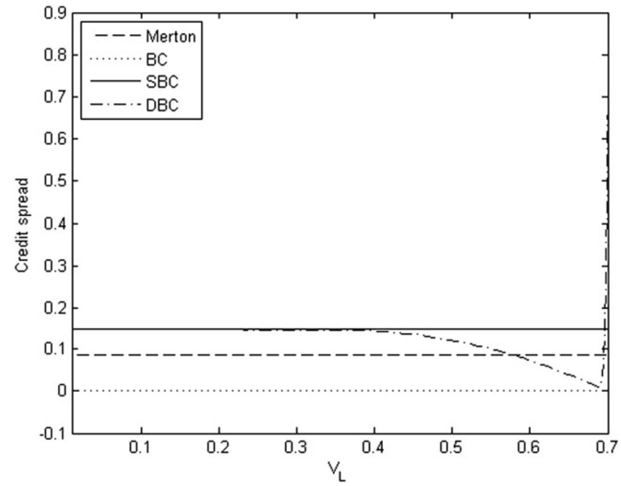

Fig. 5. Influence of V_L on the credit spread for the following four approaches: Merton (1974), Black and Cox (1976) and the Single and Double Black–Cox models proposed in this paper. The value of the volatility is $\sigma=0.5$ and the risk-free interest rate is $r=0$. The assets value at zero is $V_0=1$. The first maturity date is $T_1=1$ and the second maturity date, in the event of reorganization, is $T_2=2$. The threshold for reorganization is $V_R=0.7$. The debt amount at first maturity is $K_1=0.7$ and at second maturity is $K_2=0.7$.

denotes the current value of debtholders within each of the four approaches we study. Fig. 1 describes credit spreads versus T_1 for all four approaches. The assumptions are as in Table 1 but with $T_2=2T_1$ (this selection is based on common practices and is used only for ease of explanation). It is observed that, as maturity increases, there exists a clear difference between all four approaches, where the biggest credit spread is given by the Single Black–Cox approach, followed by Merton, Double BC and BC. Fig. 2 plots the credit spreads versus the ratio K_2/K_1 for all four approaches. The assumptions are as in Table 1. This ratio gives an idea which method might be more beneficial for debtholders. The Single Black–Cox approach is riskier when the face value of debt decreases after reorganizing; that is, it gives a higher credit spread, while being as risky as the Double Black–Cox approach when the face value of debt increases after reorganization. In Fig. 3 we describe credit spreads versus the distance between maturity times, T_2-T_1, for all four approaches. The assumptions are the same as in Table 1 but with $T_1=1$. The result is similar to those in Fig. 1 but focused on the maturity time after reorganization. Once again Single BC is the most risky approach, leading to important differences as the second maturity time increases. This implies that short waiting times between reorganization and final maturity are optimal for the SBC approach.

Next, in Figs. 4 and 5, the impact of the thresholds, V_R and V_L, on the credit spread is shown. The assumptions are as in Table 1. In Fig. 4 we plot credit spreads versus V_R for all four approaches. We must remark that V_R (V_L, V_0). It can be seen that the Single Black–Cox approach is still the most expensive in these situations, that is, it has the biggest credit spread. Fig. 5 describes credit spreads versus V_L for all four approaches. We should notice that V_L $(0,V_R)$ and, as expected, three methods are constant: Merton (1974), Black and Cox (1976) and the Single Black–Cox model.

Finally, in Figs. 6 and 7 we show the price of reorganizing for shareholders and debtholders for different levels of K_2 using the Single

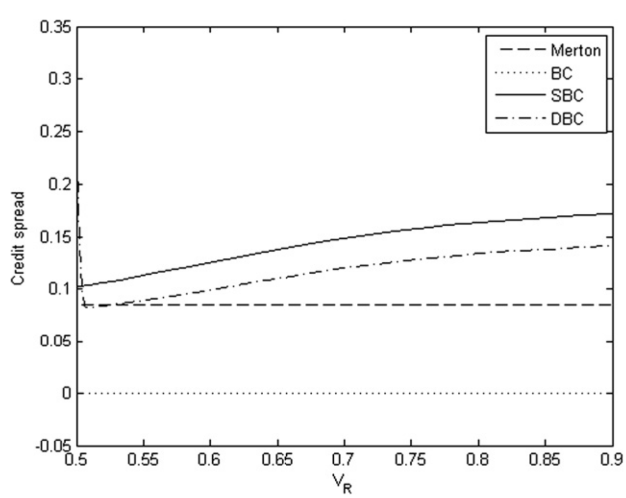

Fig. 4. Influence of V_R on the credit spread for the following four approaches: Merton (1974), Black and Cox (1976) and the Single and Double Black–Cox models proposed in this paper. The value of the volatility is $\sigma=0.5$ and the risk-free interest rate is $r=0$. The assets value at zero is $V_0=1$. The first maturity date is $T_1=1$ and the second maturity date, in the event of reorganization, is $T_2=2$. The threshold for liquidation is $V_L=0.5$. The debt amount at first maturity is $K_1=0.7$ and at second maturity is $K_2=0.7$.

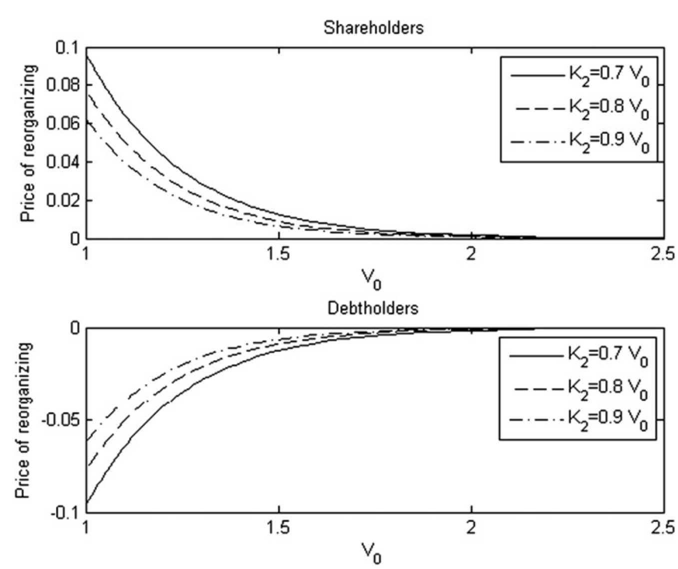

Fig. 6. Price of reorganizing for shareholders and debtholders within the Single Black–Cox approach for several choices of K_2: $0.7V_0$, $0.8V_0$ and $0.9V_0$, while $K_1=0.7V_0$. The value of the volatility is $\sigma=0.5$ and the risk-free interest rate is $r=0$. The first maturity date is $T_1=1$ and the second maturity date, in the event of reorganization, is $T_2=2$. The threshold for reorganization is $V_R=0.7V_0$ and the threshold for liquidation is $V_L=0.5V_0$.

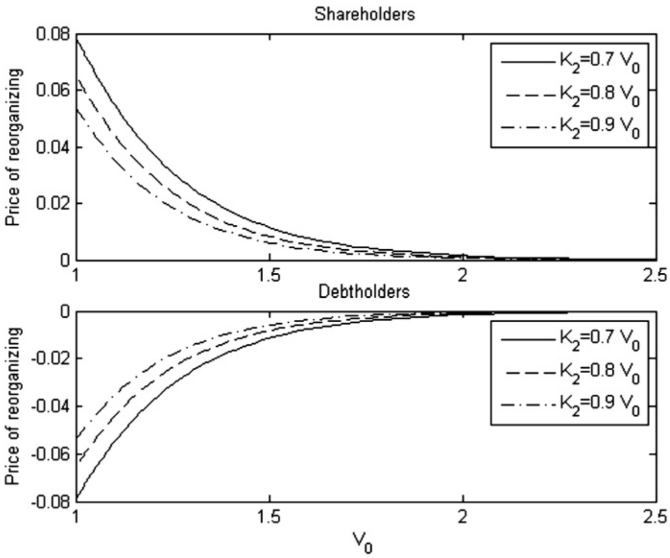

Fig. 7. Price of reorganizing for shareholders and debtholders within the Double Black–Cox approach for several choices of K_2: $0.7V_0$, $0.8V_0$ and $0.9V_0$, while $K_1=0.7V_0$. The value of the volatility is $\sigma=0.5$ and the risk-free interest rate is $r=0$. The first maturity date is $T_1=1$ and the second maturity date, in the event of reorganization, is $T_2=2$. The threshold for reorganization is $V_R=0.7V_0$ and the threshold for liquidation is $V_L=0.5V_0$.

Black–Cox and the Double Black–Cox approaches, respectively. To calculate the value for these prices we use the expressions given by Remarks 2–5. The assumptions are as in Table 1, but with $V_R=0.7V_0$, $V_L=0.5V_0$ and $K_1=0.7V_0$, since we have plotted the price of reorganizing versus the initial value of the firm, V_0. As expected, we obtain that, for both approaches, the higher the value of K_2, the less (more) beneficial the reorganization for shareholders (debtholders). Furthermore, for high values of initial firm value, the price of reorganizing is independent of the new face value of debt, and tends to zero. The reason is that for high values of V_0, there is a small probability of the firm value crossing the threshold V_R, that is, there is a small probability of carrying out a reorganization before the initial maturity of debt. Otherwise, as seen previously in Tables 1 and 2 and in the previous figures, we can notice that the benefit of a reorganization for shareholders is higher in the Single Black–Cox approach than in the Double Black–Cox. This result is due to the fact that, in the Single Black–Cox approach, there is no possibility of premature default. Consequently, we can observe that for debtholders the cost of reorganizing is lower in the Double Black–Cox approach. However, we must remember that debtholders would accept the reorganization plan if, and only if, the cost of reorganizing from their point of view is lower than the cost of liquidation, that is, if the value of debt after reorganizing is higher than the value of the payoffs they would receive in the event of the firm's liquidation.

5. Conclusions

In this paper we have developed two first-passage-time models for pricing risky debt and equity when the firm is able to carry out a reorganization through a restructuring of its debt. These two approaches generalize the Black and Cox (1976) approach, in which default is the only alternative open to a firm in the event of financial distress. In practice, reorganization consists of an alternative to liquidation as a means of maintaining the company as an operating concern.

The first model we propose is the Single Black–Cox approach. Within this approach, reorganization of the firm is begun as soon as the value of the firm crosses a lower threshold. We allow for a change both in the face value of debt and in the time to maturity. In this way, we impose no restriction on the new face value of debt; that is, it can be bigger, equal to or lower than the initial value. This is consistent with Haugen and Senbet (1978, 1988), who argue that debtholders can offer to reduce the debt claim in order to avoid bankruptcy. The Single Black–Cox analysis is more realistic than other first-passage models because it allows for reorganization before the initial maturity, without constraining the characteristics of the new debt.

Our second proposal is for the Double Black–Cox approach, which consists of a generalization of the Single Black–Cox approach, in the sense that, after reorganization, the default may occur at any time before the new maturity time. In this model, we also allow for changes in the face value of debt and in time to maturity. This seems to be a more realistic approach, because it allows default as soon as the value of the assets of the firm is too low, that is, the firm can default without waiting until the new maturity time.

Finally, we provide several numerical results with the aim of clarifying the impact that the variables and approaches have for each of the models studied. We show that within the Black–Cox framework, the value of equity is higher when there is a possibility of reorganization. This result is consistent with the evidence supported by other authors, such as Leland (1994) and Chi and Tang (2007). More specifically, we show that the benefits of reorganizing for shareholders are higher in the Single Black–Cox than in the Double Black–Cox approach, because there is no possibility of premature default and this implies a higher increase in the value of equity.

References

Abínzano, I., Navas, J.F., 2008. Pricing the equity of a firm using extendible options. Revista de Economía Financiera 15, 22–48.
Altman, E., 1984. A further empirical investigation of the bankruptcy cost question. Journal of Finance 39, 1067–1089.
Beck, T., Demirguc-Kunt, A., Levine, R., 2006. Bank concentration, competition and crises: first results. Journal of Banking and Finance 30, 1581–1603.
Branch, B., 2002. The costs of bankruptcy. A review. International Review of Financial Analysis 11, 39–57.
Black, F., Cox, J.C., 1976. Valuing corporate securities: some effects of bond indenture provisions. Journal of Finance 31, 351–367.
Bebchuk, L.A., 1988. A new approach to corporate reorganizations. Harvard Law Review 101 (4), 775–804.
Brown, D.T., 1989. Claimholder incentive conflicts in reorganization: the role of bankruptcy law. Review of Financial Studies 2 (1), 109–123.
Chi, L., Tang, T., 2007. Impact of reorganization announcements on distressed-stock re. Economic Modelling 24, 749–767.
Chatterjee, S., Dhillon, U.S., Ramírez, G.G., 1995. Coercive tender and exchange offers in high yield debt restructurings: an empirical analysis. Journal of Financial Economics 333–360.
Chatterjee, S., Dhillon, U.S., Ramírez, G.G., 1996. Resolution of financial distress: debt restructurings via Chapter 11, prepackaged bankruptcies, and workouts. Financial Management 25 (1), 5–18.
Dumitrescu, A., 2007. Valuation of defaultable bonds and debt restructuring. Journal of Corporate Finance 13, 94–111.
Ericson, J., Reneby, J., 1998. A framework for valuing corporate securities. Applied Mathematical Finance 5, 143–163.
Franks, J., Torous, W., 1989. An empirical investigation of U.S. firms in reorganization. Journal of Finance 44, 747–769.
Fisher, T., Martel, J., 2003. The effects of bankruptcy reform on the number of corporate reorganization proposals. Canadian Public Policy 29 (3), 339–350.
Fan, H., Sundaresan, S.M., 2000. Debt valuation, renegotiation, and optimal dividend policy. Review of Financial Studies 13, 1057–1099.
Francois, P., Morellec, E., 2004. Capital structure and asset prices: some effects of bankruptcy. Journal of Business 77, 387–411.
Galai, D., Raviv, A., Wiener, Z., 2007. Liquidation triggers and the valuation of equity and debt. Journal of Banking and Finance 31, 3604–3620.
Gertner, R., Scharfstein, D., 1991. A theory of workouts and the effects of reorganization law. Journal of Finance 1189–1222.
Gilson, S.C., John, K., Lang, L., 1990. Troubled debt restructurings. An empirical study of private reorganization of firms in default. Journal of Financial Economics 27, 315–353.
He, H., Keirstead, W., Rebholz, J., 1998. Double lookbacks. Mathematical Finance 8 (3), 201–228.
Haugen, R., Senbet, L., 1978. The insignificance of bankruptcy costs to the Theory of Optimal Capital Structure. Journal of Finance 33, 383–393.
Haugen, R., Senbet, L., 1988. Bankruptcy and agency costs: their significance to the Theory of Optimal Capital Structure. Journal of Financial and Quantitative Analysis 23, 27–38.
Jensen, M., 1989. Active investors, LBOs, and the privatization of bankruptcy. Journal of Applied Corporate Finance 2 (1), 35–44.
Leland, H., 1994. Corporate debt value, bond covenants, and optimal capital structure. Journal of Finance 49 (4), 1213–1252.
Longstaff, F., 1990. Pricing options with extendible maturities: analysis and applications. Journal of Finance 45, 935–957.
Merton, R.C., 1974. On the pricing of corporate debt: the risk structure of interest rate. Journal of Finance 29, 449–470.
Warner, J., 1977. Bankruptcy costs: some evidence. Journal of Finance 32 (2), 337–347.

Please cite this article as: Abínzano, I., et al., Single and Double Black–Cox: Two approaches for modelling debt restructuring, Economic Modelling (2009), doi:10.1016/j.econmod.2009.02.013