Market Crises and the 1/N Asset-Allocation Strategy

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Abstract

We consider portfolio management strategies where the investment style switches based on the value of a crisis indicator. A variety of strategies is considered in historical backtests on different datasets. Our findings show that certain simple switching strategies achieve statistically significant out-performance when compared to the equally-weighted portfolio with respect to the Sharpe ratio and Omega. In our backtest, the 1/N strategy and equal-risk contribution portfolio perform best during "normal times". On the other hand, during turbulent times, risk considerations seem to play a major role leading to minimum variance as the preferred strategy.

Keywords: Portfolio Allocation, Markov Switching, Equally Weighted, Equal-Risk Contribution

1. Introduction

Since the seminal work of Markowitz (1952), an enormous amount of research has been devoted to the problem of optimal portfolio selection. Recent contributions include the work of Maillard et al. (2010), who study *Equal-Risk Contribution* (ERC) portfolios, for which the individual risk contribution of each asset to the total portfolio risk is the same, and show that the standard deviations of these portfolios lie between those of the minimum variance portfolio and an equally-weighted portfolio (the "$1/N$" allocation). DeMiguel et al. (2009) proposed a new method of portfo-

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lio construction employing constraints on vector norms of the portfolio's position weights, enforcing diversification among the tradable assets.

Given the long history of the application of quantitative methods to portfolio selection, the prevailing trend towards naive investment strategies may be somewhat surprising. For example, Benartzi and Thaler (2001) find that more than one third of defined contribution plan investors employ the equally-weighted method when selecting an investment plan. One common feature of most advanced quantitative strategies is that they require numerous input parameters, such as expected returns, correlations and standard deviations, which need to be estimated, for example using statistical techniques and available time series data. Windcliff and Boyle (2007) show that when accounting for estimation errors the 1/N portfolio possesses some favorable characteristics with respect to robustness. DeMiguel et al. (2009) compared out-of-sample performance of many different portfolio selection methods to the $1/N$ allocation. They evaluated 14 different portfolio selection methods for seven different datasets and found the out-of-sample performance of "optimal" portfolios was rarely better than that of the 1/N strategy, stating that "for many asset allocation problems, the large error in forecasting moments of asset returns may overwhelm the gains from optimization, and so the 1/N rule may outperform the strategies from optimizing models". In their opinion the estimation errors of the parameters consume all additional performance gained through optimization. On the other hand, Kritzman et al. (2010) argue in favor of portfolio selection over the use of the 1/N strategy, claiming that poor assumptions and the use of short data windows for parameter estimation explain the under-performance of optimal portfolios in certain backtests.

The goal of this paper is to show that portfolio selection can be enhanced by applying simple but state-dependent allocation rules. In doing so two byproducts are obtained. First and foremost evidence that state dependent allocation provides a sound strategy to statistically outperform 1/N on a variety of portfolios. Secondly a confirmation of the robustness of the 1/N strategy, as in DeMiguel et al. (2009), and a method to refine it. This is due to the fact that our recommendations are to keep this strategy during normal times and switch to minimum variance during crises.

The paper proceeds by studying three allocation strategies: Minimum variance, ERC and 1/N. For Minimum variance and ERC, given a time t, the covariance is computed using the information up to t and the allocation is evaluated at $t+1$ based on the return between t and $t+1$ (out-of-sample). Three indicators which differentiate between normal times and crisis times are defined: The Turbulent Time Indicator developed by Hauptmann et al. (2012) , which is an out-of-sample indicator built using macro economic data. The Recession Indicator is used as an alternative out-of-

sample indicator available from the $FRED^{\circledR}$ database of the Federal Reserve Bank of St. Louis. The Heuristic Indicator, an in-sample indicator tailor-made from the history of financial crises. The in-sample indicator is utilized for two purposes. First to validate the hypothesis that the 1/N strategy could be out-performed in the ideal scenario that the state of the market is known in advance. Second, once this first objective is fulfilled the result from the in-sample indicator is used as benchmark for the out-of-sample indicators Turbulent Time and Recession to be compared to.
The structure of the paper is as follows: The introduction briefly summarizes the portfolio selection methods which are used throughout this work and introduces the idea of crises cycles. Section 2 presents the modeling approach and the macro-economic data utilized. In Section 3 the results of applying state dependent asset allocation are presented. Section 4 concludes.

1.1. Portfolio Strategies

One of the advantages of the 1/N strategy is its simplicity. Contrary to more advanced asset allocation methods, it is not affected by estimation error. As the number of assets N increases, risk will generally decrease due to diversification effects. As noted above, portfolio selection algorithms employing optimization often require estimated parameters, such as means and variance-covariance matrices. Estimation errors in these parameters may contaminate 'optimal' portfolios. Assuming a diffusion-based model, Merton (1980) shows that estimating expected returns from historical data is especially difficult. Errors in estimating variances and covariances are less significant, even when employing the same time series data. Consequently, optimization techniques employing estimated mean returns may be particularly suspect, and in this paper we focus solely on strategies based on risk estimates. In the sequel, the following strategies will be considered:

- Equally-weighted portfolio [1/N, naive diversification]
- Minimum-variance (Minvar) portfolio
- Equal-Risk Contribution (ERC)

The minimum-variance strategy allocates the assets into the portfolio with the smallest variance on the efficient frontier. As it only uses the covariances, the method is less vulnerable to estimation errors, as it was already pointed out by e.g. Maillard et al. (2010). The optimal weights of this strategy $\boldsymbol{x}^*_{Minvar}$ when allowing for short selling are given by:

$$\boldsymbol{x}^*_{Minvar} = \frac{\Sigma^{-1}\mathbf{1}}{\mathbf{1}'\Sigma^{-1}\mathbf{1}}. \tag{1}$$

The ERC strategy was first introduced by Qian (2005) and Neukirch (2008). Maillard et al. (2010) studied theoretical properties of the method. The idea behind equal risk contribution portfolios is to structure the portfolio in such a way that every asset in the investment universe has the same risk contribution to the total risk of the portfolio. The risk contribution of asset i is the portfolio weight of that asset, x_i, multiplied by the marginal risk of investing more in that asset: $\partial_{x_i}\sigma(x) := \partial\sigma(x)/\partial x_i$, $x = (x_1, ..., x_N)'$. In contrast to the Minvar portfolio which can consist of extreme positions in few assets, the structure of the ERC portfolio is often similar to the 1/N portfolio. For example Maillard et al. (2010) find that "[...] *minimum-variance portfolios generally suffer from the drawback of portfolio concentration*" (Maillard et al., 2010, p. 60). The ERC strategy will always invest in every asset of the investment universe, with allocation vector $\boldsymbol{x}^*_{ERC}$ implicitly given by

$$\boldsymbol{x}^*_{ERC} \in \{x \in [0,1]^N : \mathbf{1}'_N\boldsymbol{x} = 1,\ x_i \cdot \partial_{x_i}\sigma(x) = x_j \cdot \partial_{x_j}\sigma(x)\ for\ all\ i,j\}. \tag{2}$$

Since many investors are not able to short sell stocks all the implemented strategies will additionally have a long-only constraint.
In the sequel, let $\boldsymbol{R_t}$ denote the vector of excess returns (over the risk-free rate) for the N risky assets at time t. The N-dimensional vector $\boldsymbol{\mu}_t$ denotes the expected excess returns of the N risky assets at time t with sample estimate $\hat{\boldsymbol{\mu}}_t$. The $N \times N$ covariance matrix of returns is denoted by $\boldsymbol{\Sigma}_t$, with elements σ_{ij}, and sample estimate $\hat{\boldsymbol{\Sigma}}_t$. Note that only the covariance estimator but no distributional assumption is needed to calculate the ERC strategy. T is the total length of the time series and M denotes the length of the data window used for estimating the moments. The N-dimensional vector of ones is denoted as $\mathbf{1}_N$ and $\boldsymbol{I}_N$ refers to the $N \times N$ identity matrix. The vector of the portfolio weights for the N risky assets at time t is $\boldsymbol{x}_t = (x_{1,t}, \ldots, x_{N,t})'$.
In constructing parameter estimates, we apply the exponentially weighted moving average method (see, e.g. J.P.Morgan/Reuters (1996)). This estimator puts less weight on those data points farther back in time. J.P.Morgan/Reuters recommend a decay factor λ of 0.97 for monthly data. For the purpose of this work different values for $\lambda = [0.87, 0.9, 0.94, 0.97]$ were tested. The best results (in terms of the Sharpe ratio) were achieved, for the majority of databases, with the decay factor $\lambda = 0.9$ which will be used from now on.

1.2. Crises and their Identification

The salient question that arises from the work of DeMiguel et al. (2009) is whether it is possible to outperform the 1/N strategy. Kritzman et al. (2010) emphasize the importance of making the right assumptions seeking to beat this strategy. One assumption for which we will provide some evidence is that the consideration of risk is particularly important during crisis times. During normal times, the general trend of market performance is upwards, and excluding an asset due to miscalculation of risk will very likely cost performance. It is much more important to have a portfolio consisting of a broad set of assets to gain from the overall market trend. The opposite is true during a crisis. Here, the general trend is downwards, and correlations among stocks tend to increase (see Chesnay and Jondeau (2001) or Bernhart et al. (2011)). Circumstances should now favor a portfolio which consists of highly diversifying assets. The effect that estimation errors have during that time may diminish compared to the effect of a risk-optimized portfolio. In other words, during a downturn, risk matters the most and accepting estimation errors in the calculation of risk is better than complete ignorance.

To make use of the above heuristic, it is important to be able to identify turbulent times. The history of crises is long and therefore it is not surprising that there is a vast literature concerned with periods of crisis and their prediction. Kaminsky and Reinhart (1999) use 16 macro-economic and financial variables to explain the occurrence of banking and currency crises. An alternative approach was introduced by Demirguc-Kunt and Detragiache (1998). The authors use a multivariate econometric logit model to assess the probability of a banking crisis for developing and developed economies. They apply up to 13 explanatory variables which are linked to macro-economic, financial and institutional data. Kamin et al. (2001) propose a very similar method. They estimate a probit model to implement an early warning system for financial crises in 26 emerging market economies. Their explanatory variables are based on domestic and external variables as well as variables capturing external shocks.

The work in this paper builds on the class of regime-switching models, introduced by Hamilton (1989), in which the market state is determined by the value of an (unobservable) Markov chain. While Hamilton (1989) employs constant transition probabilities, Diebold et al. (1993) apply transition probabilities which change over time. Maheu and McCurdy (2000) use the Markov switching approach to distinguish between high-return stable state (bull) and low-return volatile state (bear) markets. Chesnay and Jondeau (2001) use Markov switching to split the market into times of high and low volatility to show that correlations do pick up in times of high risk. Abiad (2003) shows that regime-switching models are well suited as early warning systems for currency crises, as they are reliable in detecting crises and reducing false signals. Shiu-Sheng (2009) and Hauptmann et al. (2012) use macro-economic data to forecast the transition probabilities for a Markov switching model to detect bear markets.

2. Modeling Approach

The following sections explain the modeling approach taken in this work. We investigate whether the use of simple state-dependent asset allocation strategies performs significantly better than using only one single strategy through time.

2.1. Crisis Indicators

In this Section, we will introduce several crisis indicators which will be used in this work. It is important to note that the aim of this work is not to contribute to the long list of crises indicators, but to demonstrate the efficacy of using portfolio selection strategies that are contingent on the value of a crisis indicator.

Heuristic Indicator

The first indicator will be the Heuristic Indicator introduced by Ernst et al. (2009) which accounts for historic market crises. This Heuristic Indicator is applied to the complete time series and it is created by splitting daily stock price observations into blocks. These blocks are defined based on the underlying index reaching a 120-trading day high. After that each block is screened individually to determine if it contains a crisis. The core dates of a crisis are defined as the times when the respective prices are less than or equal to 80% of the previous 120-day high. The starting point of the crisis is set to the first time when the price falls below 90% of the previous high. The crisis' end point is defined as the date of the lowest price between the two highs. If a block does not contain any core crises, dates in the next block are considered. Looking at the performance of the S&P 500 from 1987 - 2011 (Exhibit 1) one can see that the indicator was able to identify two major crises with the burst of the Dotcom-bubble (31.08.2000 - 30.09.2002) and the Subprime-crisis (30.11.2007 - 27.02.2009) as well as two minor crises: one from 30.06.1990 - 31.10.1990 following the Gulf War and the other one from 31.07.1998 - 30.09.1998 around the time of the Russian default and the demise of Long Term Capital Management. This in-sample heuristic indicator will serve as a benchmark to test whether using state-dependent strategies really does yield improved performance, and to assess the predictive ability of other indicators.

Recession Indicator

The second indicator tries to explain financial market downturns via macro-economic recessions. This indicator is based on US recessions and can be downloaded from

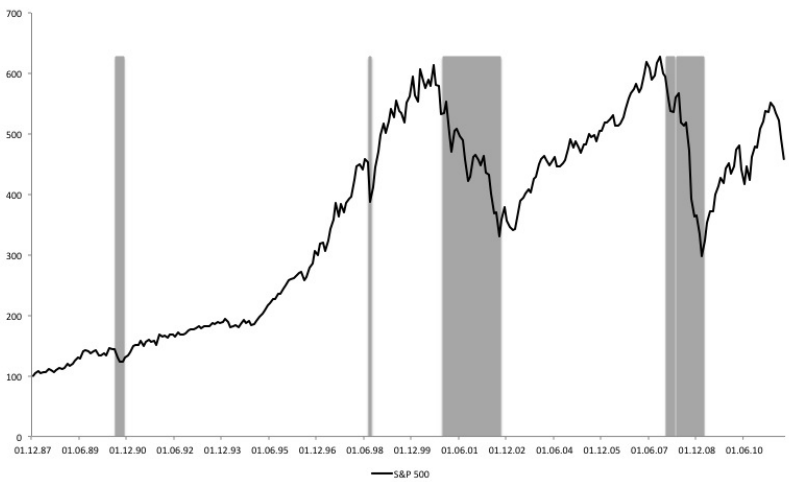

Exhibit 1: Performance of S&P 500 from 31.12.1987 - 30.09.2011. The black line represents the S&P index rebased to 100, grey shaded areas indicate heuristic capital market crises.

the $FRED^{®}$ database in the economic research section of the Federal Reserve Bank of St. Louis[6]. Applying a χ^2 test of independence based on Exhibit 2, with a null hypothesis of independence between the heuristic indicator and the Recession Indicator, yields a χ^2-statistic of $\chi^2 = 106.27$ which is higher than the critical value of 6.64 obtained for a 1% confidence level and 1 degree of freedom, implying that independence can be rejected. The differences that do exist between the heuristic crises and the recessions might come from the fact that recessions do not necessarily represent financial market crises.

Recession	Heuristic indicator: Crisis	Normal	$\sum$
Recession	24	10	*34*
Normal	15	237	*225*
$\sum$	*39*	*247*	*286*

Exhibit 2: Overlap of macro-economic recessions with heuristic capital market recessions

[6] http://research.stlouisfed.org/fred2/series/USAREC?cid=32262

Turbulent Time Indicator

Hauptmann et al. (2012) capture capital market crises by applying Markov switching models with either two or three states. The main idea of this set of models is that the market can be in one of a finite number of regimes and the unobservable state process S_t indicating the current market regime follows a Markov chain.
Within each state S_t the discrete market return process R_t is assumed to be normally distributed and can be described by a state dependent drift and volatility parameter:

$$R_t = \mu_{S_t} + \sigma_{S_t}\epsilon_t \tag{3}$$

with $\epsilon_t \sim N(0,1)$. For example, in the two-regime case, the state indicator S_t is modeled as a time-homogenous Markov chain with a fixed transition matrix:

$$\begin{bmatrix} p_{11} & 1-p_{11} \\ 1-p_{22} & p_{22} \end{bmatrix}$$

where:

$$p_{11} := \mathbb{P}(S_t = 0|S_{t-1} = 0) \text{ and } p_{22} := \mathbb{P}(S_t = 1|S_{t-1} = 1) \tag{4}$$

Hauptmann et al. (2012) use the Markov switching approach to divide the S&P 500 into two states, a calm state with normal level of volatilities ($S_t = 0$) and a turbulent state with high levels of volatility ($S_t = 1$). In a second step, they cluster the turbulent state into periods with positive expected returns and periods with negative expected returns using a second two-state Markov switching model. In the end, there are three states explaining the returns of the underlying stock market.

The parameters of the model, $\boldsymbol{\theta}$, were estimated via maximum likelihood in Hauptmann et al. (2012). After the two regimes have been identified, it is possible to compute the filtered probabilities of each state:

$$p_t^j := \mathbb{P}(S_t = j|\boldsymbol{r}_t; \hat{\boldsymbol{\theta}}_t) \tag{5}$$

with $j \in \{0,1\}$. $\boldsymbol{r}_t = (r_1, ..., r_t)$ contains the information available at time t and $\hat{\boldsymbol{\theta}}_t$ is the maximum likelihood estimate of the parameter vector based on this information. Let $S_t = 0$ indicate a calm and $S_t = 1$ a turbulent period on the stock market. Thus, $p_t^1 := \mathbb{P}(S_t = 1|\boldsymbol{r}_t; \hat{\boldsymbol{\theta}}_t)$ is an estimate for the probability of the stock market being in turbulences at time t, which only depends on information available up to time t.
To relate the filtered probabilities p_t^j from the two two-state Markov-switching mod-

els to a set of explanatory variables or macro economic variables (x_t), the authors transform the response variable with the logit function, $y_t^j = ln\left(\frac{p_t^j}{1-p_t^j}\right)$, to apply a normal linear regression model, $y_{t+1}^j = \boldsymbol{\beta^{j}}' \boldsymbol{x_t^j} + \epsilon_{t}^j,$. Therefore for any given period $[0, t]$ the state of the market at time $l+1$, $l \leq t-1$ is regressed versus macroeconomic variables observed at time l, x_l. An ARMA model for the residuals is fitted improving the estimates of the regression. This fitting allows us to forecast, hence out-of-sample, the state at $t+1$, named S_{t+1}^f using the macroeconomic information available at time t, x_t. As the time window increases the procedure is repeated entirely and state S_{t+1} is updated at time $t+1$.

Hence the probability of being in a turbulent state in the next month is forecasted using a set of economic indicators with a logit model. The probability for being in a state of high volatility, within the two state model, has to be greater than 0.5 in order to be accounted as a crisis signal. In the three-state model, additionally the probability for negative expected returns needs to be greater than 0.5. Exhibit 3

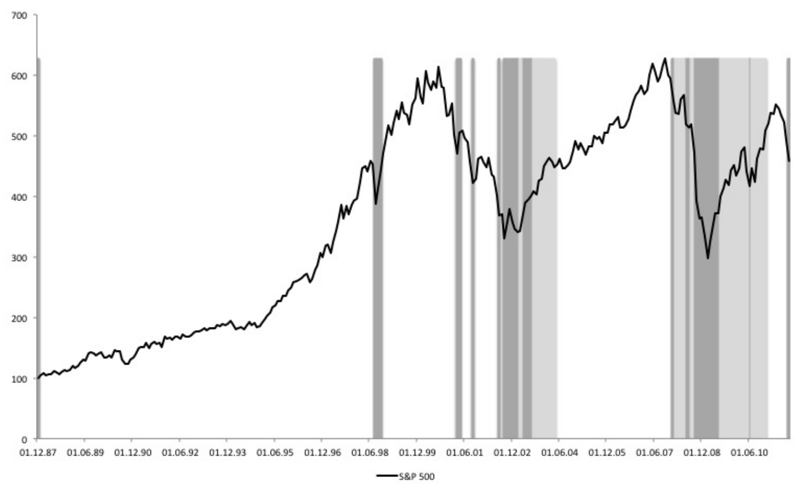

Exhibit 3: Performance of the S&P 500 from 31.12.1987 - 30.09.2011. The black line represents the S&P 500, dark grey shaded areas mark bear markets whereas light grey shaded areas mark times of turbulence with mainly positive expected returns according to the Turbulent Time Indicator of Hauptmann et al. (2012) .

shows the Turbulent Time Indicator computed using the full period under consideration. As can be seen in this Exhibit, the Turbulent Time Indicator of Hauptmann et al. is changing its signal more often than the Heuristic Indicator in Exhibit 1. A

χ^2 test of independence against the Heuristic Indicator yields a value of 32.07, which is lower than the value for the recession indicator but still above the critical value 6.64.

2.2. Macro-economic Data

The different indicators are based on publicly available (macro) economic data coming mainly from three databases. The first is the statistics portal of the OECD[7], the second is FRED® of the Federal Reserve of St. Louis[8], and the last is the Board of Governors of the Federal Reserve System[9]. Since economic data is very often published with a time lag, which sometimes might be up to one quarter, it is important to use only those data which were indeed publicly available at the point of optimization.
The recession indicator can directly be downloaded from the FRED® database and is based on OECD data. For their model, Hauptmann et al. tested 35 different indicators. They also considered transformations of those variables. In the end, 6 publicly available macroeconomic factors were selected, and augmented by 4 interaction effects among the variables.

3. Empirical Results

This section shows the empirical results of the, out-of-sample optimization using the different crisis indicators described above. To verify that the forecasts have a positive effect on the individual strategies, we first have to choose a performance measure. We focus on the Sharpe ratio and Omega, which are described below. The incorporation of economic data virtually prescribes the use of monthly or even quarterly data, since that is the regular frequency at which most of it is published. This work will use monthly data and, following the suggestion of DeMiguel et al. (2009), a window length of $M = 120$ data points equal to a 10 year estimation window. All optimization methods considered include a constraint preventing short-selling. The resulting weight vector $\hat{\boldsymbol{x}}_{t,k}$ for asset allocation strategy k is then used to invest for the upcoming month, and the portfolio return $r(\boldsymbol{x}_{t,k})$ of the corresponding strategy is calculated based on the realized excess stock returns $\boldsymbol{R}_{t+1}$ over the risk-free rate, which is taken to be the one-month Treasury bill rate obtained from the

[7]http://stats.oecd.org/mei/default.asp?lang=e
[8]http://research.stlouisfed.org/fred2/
[9]http://www.federalreserve.gov/releases/h15/data.htm

Fama French website:

$$r\left(\boldsymbol{x}_{t,k}\right) = \boldsymbol{x}_{t,k}{}'\boldsymbol{R}_{t+1} \tag{6}$$

By employing a rolling estimation window of length M, we obtain k time series of portfolio returns $r\left(\boldsymbol{x}_{t,k}\right)$ with length T-M.
Performance is tested using market data from several different sources. In total six multi-factor datasets are considered from the webpage of Fama and French[10]. Additionally we used three international stock datasets provided by MSCI[11]. These datasets include the MSCI indices for Canada, France, Germany, Italy, Japan, Switzerland, the United Kingdom, the United States and the World Aggregate. The last dataset consists of the S&P 500 index and an US Government Bond Index. A detailed description of the datasets and the methodology used to build them can be found in the Appendix. Exhibit 4 briefly summarizes all the datasets that were used, and lists the numbers of risky assets and the maximum available time horizon for each. Due to limitied availability of macro-economic data the Turbulent Time Indicator by Hauptmann et al. (2012) cannot reach further back than December 1987. To ensure comparability across the different indicators, we therefore consider monthly data from 12.1987 until 09.2011 as was done by Hauptmann et al.

Data Set		# of Risky Assets	Time Horizon
Industry portfolio	Dataset 1	10	07.1927 - 07.2012
US Market, HML, SMB & Book-to-Market	Dataset 2	23	07.1926 - 07.2012
Portfolios formed on Book-to-Market	Dataset 3	5	07.1926 - 07.2012
Size and Momentum	Dataset 4	6	01.1927 - 07.2012
US Growth Portfolio	Dataset 5	5	07.1926 - 07.2012
US Value Portfolio	Dataset 6	5	07.1926 - 07.2012
International Stock Indices	Dataset 7	9	12.1969 - 10.2012
International Growth Portfolio	Dataset 8	9	12.1974 - 10.2012
International Value Portfolio	Dataset 9	9	12.1974 - 10.2012
S&P 500 and US Government Bond	Dataset 10	2	12.1987 - 10.2012

Exhibit 4: Datasets employed in the historical backtests.

3.1. Performance measurement

The performance measures employed in this paper are the Sharpe and Omega ratios. The Sharpe ratio was first introduced by Treynor (1965) and then applied

[10]http://mba.tuck.dartmouth.edu/pages/faculty/ken.french/data_library.html
[11]http://www.msci.com/products/indices/performance.html

and adjusted by Sharpe (1966) and Sharpe (1970).

$$\widehat{SR}_k = \frac{\hat{\mu}_k}{\hat{\sigma}_k} \tag{7}$$

With $\hat{\mu}_k$ and $\hat{\sigma}_k$ being the sample mean and standard deviation of the excess return respectively, using the entire time series with length T-M for allocation strategy k. The Omega ratio was introduced by Shadwick and Keating (2002). This performance measure is very well suited for non normal distributions since it also incorporates higher moments next to mean and standard deviation. This is especially useful for asymmetric return distributions. The Omega for strategy k, Ω_k, is defined as the ratio of the positive to the negative expected value of excess returns with respect to a chosen benchmark. The sample equivalent for strategy k based on observed data will be denoted by $\hat{\Omega}_k$:

$$\hat{\Omega}_k = \frac{E\left[\hat{\mu}_k^+\right]}{E\left[\hat{\mu}_k^-\right]}, \tag{8}$$

with

$$E\left[\hat{\mu}_k^{\pm}\right] = \frac{1}{T-M} \sum_{s=1}^{T-M} \max\left\{\pm r\left(\boldsymbol{x}_{s,k}\right), 0\right\}$$

In the given case, the benchmark is taken to be the risk-free rate (i.e. the one-month Treasury bill rate). After calculating the performance measures, it is important to assess whether the different strategies performed significantly better than the 1/N strategy. The null hypothesis which shall be rejected is given by:

$$H_0 : \widehat{SR}_{\frac{1}{N}} = \widehat{SR}_k \tag{9}$$

$$H_0 : \Omega_{\frac{1}{N}} = \Omega_k \tag{10}$$

To test for significance in the difference of Sharpe ratios, this paper follows the method suggested by Jobson and Korkie (1981) to calculate z scores, including the correction suggested by Memmel (2003). For testing the hypothesis that the Omegas for two strategies are the same, we follow the methodology of Schmid and Schmidt (2007). They present an estimator for the variance of the difference of two Omega ratios $\hat{\sigma}^2_{\Omega_{1/N}-\Omega_k}$. According to Schmid and Schmidt the z-value is then given by:

$$z_{\Omega_{1/N}-\Omega_k} = \frac{\sqrt{T-M}\left(\hat{\Omega}_{1/N} - \hat{\Omega}_k\right)}{\hat{\sigma}_{\Omega_{1/N}-\Omega_k}} \sim N(0,1). \tag{11}$$

The null hypothesis: $\Omega_k = \Omega_{1/N}$ can be rejected if $\left|z_{\Omega_{1/N}-\Omega_k}\right| > \Phi^{-1}\left(1-\frac{\alpha}{2}\right)$, with α being the prescribed confidence level.

3.2. Results for the Heuristic Indicator

In the following sections we investigate the performance of switching strategies in a series of historical backtests using the Sharpe ratio and Omega described above. We begin by demonstrating that foreknowledge of the different market states offers a significant advantage, for which purpose we employ the heuristic indicator for capital market recessions constructed with S&P 500 data using the complete data set.

Exhibit 5 shows the performance of the different strategies when splitting the time series according to the heuristic indicator. For all datasets, the 1/N strategy shows the highest returns during normal times. In turbulent times, where risk considerations matter the most, it is the minimum variance portfolio that shows the smallest losses. Even though the Minvar method is prone to estimation errors, the 1/N strategy performs worse in bearish markets. Exhibit 5 gives some suggestion as to which strategy should be used given the economic situation. We now consider switching strategies, where the investment portfolio employed in the period $[t, t+1)$ depends on the value of the heuristic indicator at $t+1$. For the purpose of completeness, for this indicator and all other indicators, we consider all 6 possible switching strategies; 1/N & Minvar as well as 1/N & ERC. Then ERC & Minvar and ERC & 1/N. And the combination of Minvar & 1/N and Minvar & ERC. In this notation the first term denotes the strategy applied during normal times while the second term denotes the strategy during crisis.

Exhibit 6 demonstrates that it is possible to find switching strategies that perform significantly better than the 1/N strategy. Combinations of 1/N & Minvar or ERC & Minvar consistently outperform a straight 1/N allocation in terms of Sharpe ratios and Omega values. Furthermore, when combining 1/N & ERC it is possible to be significantly better than 1/N alone.

The above analysis relied on knowledge of the heuristic indicator, and consequently assumes some knowledge about future returns. Next, we repeat the analysis using the recession indicator, which can be constructed based on public data at the time of the investment decision, and the Turbulent Time Indicator, for which the probabilities

		Normal			Turbulent		
		1/N	ERC	Minvar	1/N	ERC	Minvar
Dataset 1	Mean	**1.56**	1.51	1.32	-3.10	-2.89	**-2.07**
	Std	3.43	3.19	3.02	5.57	5.12	4.28
Dataset 2	Mean	**1.54**	1.42	0.38	-2.59	-2.02	**-0.11**
	Std	4.01	3.52	1.49	7.05	6.35	2.47
Dataset 3	Mean	**1.58**	**1.58**	1.45	-3.09	-3.06	**-2.57**
	Std	3.54	3.50	3.30	6.03	6.00	5.71
Dataset 4	Mean	**1.63**	1.59	1.52	-3.38	-3.24	**-2.65**
	Std	4.21	4.00	3.59	7.26	6.80	5.71
Dataset 5	Mean	**1.57**	**1.57**	1.53	-3.07	-3.04	**-2.76**
	Std	3.62	3.60	3.57	5.81	5.80	5.68
Dataset 6	Mean	**1.63**	1.60	1.53	-3.19	-3.14	–**2.67**
	Std	3.76	3.69	3.51	6.68	6.63	6.58
Dataset 7	Mean	**1.26**	1.25	1.10	-4.04	-3.94	**-3.60**
	Std	3.83	3.67	3.44	6.79	6.55	5.72
Dataset 8	Mean	**1.17**	1.15	1.05	-4.16	-3.87	**-3.12**
	Std	3.98	3.76	3.41	7.14	6.54	5.40
Dataset 9	Mean	**1.33**	**1.33**	1.26	-4.17	-4.04	**-3.28**
	Std	3.94	3.76	3.63	6.98	6.74	5.71
Dataset 10	Mean	**0.88**	0.71	0.59	-1.44	-0.48	**0.05**
	Std	2.21	1.73	1.58	3.16	2.38	2.22

Exhibit 5: State dependent means and standard deviations of returns for the three different strategies. For distinguishing between the states, the heuristic indicator is applied. The allocations with the highest mean returns per state are displayed in bold figures. All numbers are displayed in percentages and on a monthly basis.

of the two market states can be forecasted using economic variables.

3.3. Results for the Recession Indicator

In this section, we investigate switching strategies in which the portfolio allocation method depends on the value of the recession indicator. Exhibit 2 demonstrated that economic recessions mainly coincide with financial market crises. However, owing to the time lag in the publication of economic data, we must be careful to ensure that in our backtests data is only used when it was publicly available. Hence the results are out-of-sample. For that purpose, the economic dataset is shifted by two months to account for the time lag in the calculation. We also note that the recession indicator focuses exclusively on economic activity in the United States. Given that most of the assets in our investment universes are US indices, this should not lead to a significant bias in the results. Only three datasets (those for the international stock indices) are not exclusively US based. For the international stock portfolios it would be possible to use a global recession indicator. However such an indicator would also not be an adequate aggregation, since only a small fraction of stock indices of the developed

world were used.
Results for backtests on all possible switching strategies involving the basic allocation methods and the value of the recession indicator are presented in Exhibit 7. When applying this indicator only 4 datatsets show significantly better Sharpe ratios than the 1/N strategy. Comparing the Omega measures, 5 exhibit better performance than the equally-weighted strategy. Even though economic recessions might be a good indicator for capital market crises, using this indicator out-of-sample comes with a significant loss in predictive power. This is because knowing whether there is an economic recession is very often only possible from an ex-post perspective, when most of the losses have already happened and risk is fully priced in. The same is also true for the heuristic indicator (when it is constructed using lagged data). It is important to apply a crisis indicator like the Turbulent Time Indicator from Hauptmann et al. (2012) which directly aims at financial markets and at the same time has a predictive character.

3.4. Results for the Turbulent Time Indicator

As noted above, the Turbulent Time Indicator of Hauptmann et al. is a pure financial market indicator. The probabilities for the different states are based on the movements of the S&P 500. These probabilities, which estimate the chance that the market will be in a calm or turbulent state next period, are forecasted using economic data available at the current time.
Even though the χ^2 statistic for the 3 state indicator was not as high as for the recession indicator, the backtest results for switching strategies using this indicator are better than when using the OECD data, as can be seen in Exhibit 8. In only three datasets did it not lead to significant outperformance of the 1/N allocation. For seven datasets it was possible to achieve significantly better results for both Sharpe ratio and Omega values, when using a combination of 1/N & Minvar. The failure to outperform in the other datasets might be due to the fact that the Turbulent Time Indicator is fitted to the S&P 500 index and not to any subset of it.

4. Conclusion

This work provides several results regarding portfolio allocation based on out-of-sample backtest analysis of well known market portfolios. In certain historical backtests, we find that when distinguishing between calm and turbulent times it was possible to perform significantly better than the equally-weighted portfolio. Moreover, during normal times, the 1/N strategy generally outperforms most other allocation methods, refining the results of DeMiguel et al. (2009). Intuitively, it appears

that during "normal" times, considerations regarding estimation error override those of risk minimization, while in turbulent times the opposite holds.
During turbulent times, risk considerations seem to play a major role. From our results we find evidence that this is the time when investors should concentrate on positions which offer the lowest risk. It appears better to accept the presence of estimation errors than to ignore completely the potential for risk reduction due to optimization. This can be seen when looking at the results of Exhibit 6, where the majority of Sharpe ratios of strategies which were using the Minvar method instead of 1/N in turbulent times were higher than those of the 1/N approach. Using different strategies during calm and turbulent times does offer significantly better Sharpe ratios than the 1/N method. The two new portfolio allocation strategies which showed superior performance are:

	Calm	Turbulent
Strategy 1	1/N	Minvar
Strategy 2	ERC	Minvar

Exhibit 9: Switching strategies that consistently outperformed 1/N in historical backtests.

The second interesting result of this paper is that economic data can be used to distinguish between the different market states. When employing the US recession indicator, the results support a certain linkage between the real economy and the financial markets, although this linkage is not perfect. In this paper, economic data was used to forecast the probabilities of turbulent times. The regime-switching indicator proved to offer significantly better Sharpe ratios and Omega values for many datasets. We also demonstrated that the model introduced by Hauptmann et al. (2012) offers a useful indicator of capital market crises. In future research it would be interesting to investigate whether performance increases further when applying the Markov switching method directly to the different data subsets, markets or asset classes which were represented by the portfolios. Since the model of Hauptmann et al. can distinguish between three states, introducing strategies using a combination of three allocation methods would also be interesting.

Industry portfolio (Dataset 1)
The first dataset consists of ten industry indices and was obtained from Kenneth French's website. These indices are constructed by assigning each stock of NYSE, AMEX and NASDAQ to one of ten industry portfolios. The attribution is done

based on the four digit SIC code of every asset.

US Market, HML, SMB and portfolios formed on Size- and Book-to-Market (Dataset 2)
This is the Fama/French dataset formed on the factors size and book-to-market and consists of 25 return series. Following Wang (2005) and DeMiguel et al. (2009) we also exclude the 5 series with the largest companies leaving 20 return series in total. The Fama/French datasets called SMB (Small Minus Big) dataset, HML (High Minus Low) US Market are added.

Portfolios formed on Book-to-Market (Dataset 3)
This dataset uses the the book-to-market equity ratio as the only grouping criterion. All stocks of the NYSE, AMEX and NASDAQ are included and therefore this dataset largely consists of the dame assets as the previous two datasets. The stocks are grouped in value-weighted quintiles of stocks having a positive book equity at time t.

Dataset formed on Size and Momentum (Dataset 4)
Following DeMiguel et al. (2009), we consider a dataset in which the return series are formed on Size and Momentum criteria. To create those portfolios, the stocks of the NYSE, AMEX and NASDAQ are first split into small and large companies using the monthly median NYSE market equity as the breaking point. After that, the stocks are additionally divided into three momentum categories, based on the performance over the previous 12 months. The break points for momentum are the monthly 30^{th} and 70^{th} percentile of NYSE returns.

US Growth and Value Portfolios (Datasets 5 and 6)
An additional possibility to distinguish assets is to split stocks into growth and value categories. As discussed by Fama and French (1998), there are several indicators one can use to assess whether a stock is a growth stock or a value stock. Herein, the Fama/French universe is divided into book-to-market equity deciles. The "*US Growth*" dataset (*Dataset 5*) formed on that basis consists of the five return series with the relatively low book-to-market equity ratio. The "*US Value*" dataset (*Dataset 6*) accordingly consists of the five indices with relatively high book-to-market ratios.

International Stock Indices (Datasets 7, 8, and 9)
These datasets are constructed using nine international stock indices. As in the pa-

per of DeMiguel et al. (2009), the MSCI indices for Canada, France, Germany, Italy, Japan, Switzerland, the United Kingdom, the United States and the World were considered. As with the US only datasets, we may also distinguish between growth and value stocks. The portfolio without distinguishing between the different styles will be called *Dataset 7*, the "International Growth" portfolio is *Dataset 8*, and the "International Value" portfolio will be called *Dataset 9*.

Portfolio Consisting of S&P 500 and US Government Bond Index (Dataset 10)
The datasets described above consist of 100% equity investments. This dataset consists of an equity component, the S&P 500 index, and a fixed income component, the JPM Global Aggregated Bond Index, denominated in US$.

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		$\frac{1}{N}$	ERC	Minvar	$\frac{1}{N}$/Minvar	ERC/Minvar	Minvar/$\frac{1}{N}$	Minvar/ERC	ERC/$\frac{1}{N}$	$\frac{1}{N}$/ERC
Dataset 1	SR	0.1490	0.1563*	0.1599	0.1997***	0.1985***	0.1073**	0.1183*	0.1448	0.1599***
	p-value	-	(0.09)	(0.34)	(0.00)	(0.00)	(0.04)	(0.10)	(0.20)	(0.00)
	Ω	1.46	1.49**	1.50	1.65***	1.64***	1.33**	1.36*	1.45	1.50***
	p-value	-	(0.05)	(0.31)	(0.00)	(0.00)	(0.03)	(0.08)	(0.23)	(0.00)
Dataset 2	SR	0.1393	0.1523**	-0.0025***	0.2577***	0.2609***	-0.1107***	-0.0935***	0.1273**	0.1623***
	p-value	-	(0.04)	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)	(0.04)	(0.00)
	Ω	1.43	1.49***	0.99***	1.93***	1.96***	0.69***	0.74***	1.40*	1.51***
	p-value	-	(0.01)	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)	(0.06)	(0.00)
Dataset 3	SR	0.1482	0.1486	0.1424	0.1627***	0.1620***	0.1276**	0.1288**	0.1474	0.1494***
	p-value	-	(0.38)	(0.31)	(0.00)	(0.01)	(0.02)	(0.03)	(0.20)	(0.01)
	Ω	1.46	1.46	1.44	1.51***	1.51***	1.40**	1.40**	1.46	1.46***
	p-value	-	(0.33)	(0.33)	(0.00)	(0.00)	(0.03)	(0.04)	(0.18)	(0.00)
Dataset 4	SR	0.1252	0.1289	0.1521*	0.1559***	0.1542***	0.1173	0.1246	0.1222	0.1317***
	p-value	-	(0.16)	(0.07)	(0.00)	(0.00)	(0.32)	(0.49)	(0.19)	(0.00)
	Ω	1.38	1.39	1.48**	1.49***	1.48***	1.36	1.38	1.37	1.40***
	p-value	-	(0.19)	(0.05)	(0.00)	(0.00)	(0.35)	(0.48)	(0.15)	(0.00)
Dataset 5	SR	0.1446	0.1465**	0.1507	0.1569***	0.1577***	0.1383	0.1395	0.1453	0.1457***
	p-value	-	(0.04)	(0.31)	(0.00)	(0.00)	(0.29)	(0.32)	(0.22)	(0.01)
	Ω	1.44	1.45***	1.47	1.49***	1.49***	1.42	1.43	1.45	1.45***
	p-value	-	(0.01)	(0.25)	(0.00)	(0.00)	(0.27)	(0.31)	(0.20)	(0.00)
Dataset 6	SR	0.1431	0.1425	0.1503	0.1616***	0.1589***	0.1312	0.1335	0.1403**	0.1453***
	p-value	-	(0.35)	(0.28)	(0.00)	(0.01)	(0.14)	(0.19)	(0.03)	(0.00)
	Ω	1.46	1.46	1.48	1.53***	1.52***	1.41	1.42	1.45**	1.47***
	p-value	-	(0.26)	(0.31)	(0.00)	(0.00)	(0.11)	(0.15)	(0.02)	(0.00)
Dataset 7	SR	0.0466	0.0492	0.0346	0.0626**	0.0623*	0.0190*	0.0222	0.0458	0.0499***
	p-value	-	(0.25)	(0.30)	(0.05)	(0.06)	(0.09)	(0.12)	(0.42)	(0.01)
	Ω	1.13	1.14	1.10	1.18***	1.18**	1.05**	1.06*	1.13	1.14***
	p-value	-	(0.21)	(0.23)	(0.01)	(0.02)	(0.04)	(0.06)	(0.42)	(0.00)
Dataset 8	SR	0.0262	0.0335	0.0412	0.0606**	0.0596**	0.0052	0.0147	0.0239	0.0355**
	p-value	-	(0.14)	(0.30)	(0.02)	(0.03)	(0.17)	(0.31)	(0.33)	(0.03)
	Ω	1.07	1.09**	1.11	1.17***	1.17***	1.01*	1.04	1.07	1.10***
	p-value	-	(0.05)	(0.22)	(0.00)	(0.00)	(0.08)	(0.23)	(0.27)	(0.00)
Dataset 9	SR	0.0552	0.0598	0.0765	0.0864***	0.0877***	0.0443	0.0486	0.0554	0.0594***
	p-value	-	(0.17)	(0.22)	(0.00)	(0.00)	(0.33)	(0.40)	(0.48)	(0.00)
	Ω	1.16	1.17	1.22	1.25***	1.25***	1.13	1.14	1.16	1.17***
	p-value	-	(0.13)	(0.15)	(0.00)	(0.00)	(0.28)	(0.35)	(0.49)	(0.00)
Dataset 10	SR	0.0583	0.1157**	0.1414*	0.2135***	0.1835***	-0.0356**	0.0712	0.0078***	0.1559***
	p-value	-	(0.04)	(0.09)	(0.00)	(0.00)	(0.02)	(0.40)	(0.01)	(0.00)
	Ω	1.16	1.36***	1.45***	1.73***	1.62***	0.91***	1.21	1.02***	1.50***
	p-value	-	(0.00)	(0.01)	(0.00)	(0.00)	(0.00)	(0.31)	(0.00)	(0.00)

Exhibit 6: Empirical Sharpe ratios and Omega values and their p-values for all datasets using the heuristic indicator. *** Indicates a significantly different performance than the 1/N method for the 1% confidence level, ** for the 5% confidence level and * for the 10% confidence level

		$\frac{1}{N}$	ERC	Minvar	$\frac{1}{N}$/Minvar	ERC/Minvar	Minvar/$\frac{1}{N}$	Minvar/ERC	ERC/$\frac{1}{N}$	$\frac{1}{N}$/ERC
Dataset 1	SR	0.1490	0.1563*	0.1599	0.1732*	0.1754**	0.1339	0.1407	0.1494	0.1554*
	p-value	-	(0.09)	(0.34)	(0.06)	(0.05)	(0.23)	(0.35)	(0.46)	(0.06)
	Ω	1.46	1.49**	1.50	1.55**	1.56**	1.41	1.44	1.47	1.49**
	p-value	-	(0.05)	(0.31)	(0.03)	(0.02)	(0.21)	(0.33)	(0.38)	(0.03)
Dataset 2	SR	0.1393	0.1523**	-0.0025***	0.1394	0.1453	0.0374	0.0451***	0.1413	0.1488**
	p-value	-	(0.04)	(0.00)	(0.50)	(0.40)	(0.00)	(0.00)	(0.38)	(0.04)
	Ω	1.43	1.49***	0.99***	1.46	1.49	1.12***	1.14***	1.45	1.46***
	p-value	-	(0.01)	(0.00)	(0.35)	(0.21)	(0.00)	(0.00)	(0.16)	(0.01)
Dataset 3	SR	0.1482	0.1486	0.1424	0.1458	0.1459	0.1448	0.1451	0.1483	0.1484
	p-value	-	(0.38)	(0.31)	(0.36)	(0.37)	(0.36)	(0.37)	(0.45)	(0.36)
	Ω	1.46	1.46	1.44	1.45	1.45	1.46	1.46	1.46	1.46
	p-value	-	(0.33)	(0.33)	(0.27)	(0.28)	(0.45)	(0.46)	(0.39)	(0.36)
Dataset 4	SR	0.1252	0.1289	0.1521*	0.1357	0.1369*	0.1380	0.1394	0.1280	0.1259
	p-value	-	(0.16)	(0.07)	(0.17)	(0.09)	(0.19)	(0.17)	(0.19)	(0.39)
	Ω	1.38	1.39	1.48**	1.40	1.42	1.45*	1.45*	1.39	1.38
	p-value	-	(0.19)	(0.05)	(0.24)	(0.14)	(0.09)	(0.09)	(0.13)	(0.43)
Dataset 5	SR	0.1446	0.1465**	0.1507	0.1469	0.1482	0.1484	0.1490	0.1458*	0.1452
	p-value	-	(0.04)	(0.31)	(0.38)	(0.32)	(0.34)	(0.31)	(0.07)	(0.16)
	Ω	1.44	1.45***	1.47	1.45	1.45	1.46	1.46	1.45**	1.45
	p-value	-	(0.01)	(0.25)	(0.41)	(0.33)	(0.24)	(0.22)	(0.02)	(0.17)
Dataset 6	SR	0.1431	0.1425	0.1503	0.1428	0.1427	0.1503	0.1499	0.1430	0.1426
	p-value	-	(0.35)	(0.28)	(0.49)	(0.48)	(0.21)	(0.23)	(0.45)	(0.34)
	Ω	1.46	1.46	1.48	1.45	1.45	1.49	1.49	1.46	1.46
	p-value	-	(0.26)	(0.31)	(0.34)	(0.34)	(0.12)	(0.15)	(0.46)	(0.17)
Dataset 7	SR	0.0466	0.0492	0.0346	0.0606	0.0615	0.0209*	0.0228	0.0471	0.0487
	p-value	-	(0.25)	(0.30)	(0.12)	(0.12)	(0.09)	(0.11)	(0.45)	(0.14)
	Ω	1.13	1.14	1.10	1.17*	1.18*	1.06**	1.06**	1.13	1.14*
	p-value	-	(0.21)	(0.23)	(0.07)	(0.07)	(0.03)	(0.05)	(0.43)	(0.07)
Dataset 8	SR	0.0262	0.0335	0.0412	0.0612**	0.0597**	0.0049	0.0145	0.0237	0.0357**
	p-value	-	(0.14)	(0.30)	(0.04)	(0.05)	(0.14)	(0.28)	(0.29)	(0.04)
	Ω	1.07	1.09**	1.11	1.17***	1.17***	1.01**	1.04	1.07	1.10***
	p-value	-	(0.05)	(0.22)	(0.00)	(0.00)	(0.07)	(0.21)	(0.24)	(0.00)
Dataset 9	SR	0.0552	0.0598	0.0765	0.0828**	0.0865***	0.0473	0.0493	0.0578	0.0572
	p-value	-	(0.17)	(0.22)	(0.02)	(0.01)	(0.037)	(0.40)	(0.29)	(0.17)
	Ω	1.16	1.17	1.22	1.24***	1.25***	1.14	1.14	1.17	1.17*
	p-value	-	(0.13)	(0.15)	(0.00)	(0.00)	(0.32)	(0.36)	(0.25)	(0.09)
Dataset 10	SR	0.0583	0.1157**	0.1414*	0.1300**	0.1522**	0.0491	0.1040	0.0611	0.1021***
	p-value	-	(0.04)	(0.09)	(0.03)	(0.02)	(0.42)	(0.18)	(0.46)	(0.01)
	Ω	1.16	1.36***	1.45***	1.39***	1.49***	1.15	1.32**	1.18	1.30
	p-value	-	(0.00)	(0.01)	(0.00)	(0.00)	(0.42)	(0.05)	(0.34)	(0.00)

Exhibit 7: Empirical Sharpe ratios and Omega values and their p-values for all datasets using the US recession indicator out-of-sample. *** Indicates a significantly different performance than the 1/N method for the 1% confidence level, ** for the 5% confidence level and * for the 10% confidence level.

		$\frac{1}{N}$	ERC	Minvar	$\frac{1}{N}$/Minvar	ERC/Minvar	Minvar/$\frac{1}{N}$	Minvar/ERC	ERC/$\frac{1}{N}$	$\frac{1}{N}$/ERC
Dataset 1	SR	0.1490	0.1563*	0.1599	0.1769***	0.1791***	0.1308	0.1377	0.1491	0.1557***
	p-value	-	(0.09)	(0.34)	(0.01)	(0.01)	(0.21)	(0.31)	(0.49)	(0.01)
	Ω	1.46	1.49**	1.50	1.55***	1.57***	1.41	1.43	1.47	1.48***
	p-value	-	(0.05)	(0.31)	(0.01)	(0.01)	(0.22)	(0.31)	(0.31)	(0.01)
Dataset 2	SR	0.1393	0.1523**	-0.0025***	0.2030***	0.2093***	-0.0372***	-0.0297***	0.1359	0.1541***
	p-value	-	(0.04)	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)	(0.31)	(0.00)
	Ω	1.43	1.49***	0.99***	1.66***	1.70***	0.88***	0.91***	1.43	1.48***
	p-value	-	(0.01)	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)	(0.42)	(0.00)
Dataset 3	SR	0.1482	0.1486	0.1424	0.1569**	0.1566*	0.1340*	0.1347*	0.1479	0.1489*
	p-value	-	(0.38)	(0.31)	(0.05)	(0.06)	(0.08)	(0.09)	(0.36)	(0.08)
	Ω	1.46	1.46	1.44	1.48*	1.48*	1.42	1.42	1.46	1.46
	p-value	-	(0.33)	(0.33)	(0.08)	(0.08)	(0.11)	(0.12)	(0.46)	(0.13)
Dataset 4	SR	0.1252	0.1289	0.1521*	0.1357	0.1389*	0.1379	0.1401	0.1274	0.1265
	p-value	-	(0.16)	(0.07)	(0.17)	(0.10)	(0.19)	(0.16)	(0.22)	(0.32)
	Ω	1.38	1.39	1.48**	1.40	1.42	1.45*	1.45*	1.39	1.38
	p-value	-	(0.19)	(0.05)	(0.27)	(0.17)	(0.08)	(0.08)	(0.11)	(0.46)
Dataset 5	SR	0.1446	0.1465**	0.1507	0.1419	0.1442	0.1534	0.1529	0.1469***	0.1441
	p-value	-	(0.04)	(0.31)	(0.32)	(0.47)	(0.20)	(0.22)	(0.01)	(0.16)
	Ω	1.44	1.45***	1.47	1.43	1.44	1.48	1.48	1.45***	1.44
	p-value	-	(0.01)	(0.25)	(0.28)	(0.48)	(0.15)	(0.16)	(0.00)	(0.11)
Dataset 6	SR	0.1431	0.1425	0.1503	0.1496	0.1480	0.1436	0.1447	0.1414*	0.1442
	p-value	-	(0.35)	(0.28)	(0.21)	(0.27)	(0.48)	(0.44)	(0.09)	(0.16)
	Ω	1.46	1.46	1.48	1.47	1.47	1.47	1.47	1.45	1.46
	p-value	-	(0.26)	(0.31)	(0.32)	(0.39)	(0.41)	(0.39)	(0.12)	(0.34)
Dataset 7	SR	0.0466	0.0492	0.0346	0.0679***	0.0674**	0.0145*	0.0177*	0.0456	0.0502**
	p-value	-	(0.25)	(0.30)	(0.01)	(0.02)	(0.06)	(0.08)	(0.40)	(0.02)
	Ω	1.13	1.14	1.10	1.19***	1.19***	1.04**	1.05**	1.13	1.14***
	p-value	-	(0.21)	(0.23)	(0.00)	(0.00)	(0.02)	(0.03)	(0.40)	(0.01)
Dataset 8	SR	0.0262	0.0335	0.0412	0.0472**	0.0529***	0.0181	0.0212	0.0303	0.0291
	p-value	-	(0.14)	(0.30)	(0.03)	(0.01)	(0.38)	(0.42)	(0.27)	(0.13)
	Ω	1.07	1.09**	1.11	1.13***	1.15***	1.05	1.06	1.08	1.08*
	p-value	-	(0.05)	(0.22)	(0.00)	(0.00)	(0.31)	(0.38)	(0.16)	(0.06)
Dataset 9	SR	0.0552	0.0598	0.0765	0.0895***	0.0905***	0.0423	0.0466	0.0553	0.0596**
	p-value	-	(0.17)	(0.22)	(0.00)	(0.00)	(0.30)	(0.36)	(0.49)	(0.04)
	Ω	1.16	1.17	1.22	1.26***	1.26***	1.12	1.13	1.16	1.17
	p-value	-	(0.13)	(0.15)	(0.00)	(0.00)	(0.23)	(0.30)	(0.49)	(0.02)
Dataset 10	SR	0.0583	0.1157**	0.1414*	0.1594***	0.1615***	0.0264	0.0956	0.0448	0.1212***
	p-value	-	(0.04)	(0.09)	(0.01)	(0.01)	(0.23)	(0.23)	(0.28)	(0.00)
	Ω	1.16	1.36***	1.45***	1.49***	1.53***	1.08	1.29*	1.13	1.36***
	p-value	-	(0.00)	(0.01)	(0.00)	(0.00)	(0.14)	(0.09)	(0.26)	(0.00)

Exhibit 8: Empirical Sharpe ratios and Omega values and their p-values for all datasets using the Turbulent Time Indicator (3 states). *** Indicates a significantly different performance than the 1/N method for the 1% confidence level, ** for the 5% confidence level and * for the 10% confidence level.