Stochastic Correlation in Risk Analytics: a Financial Perspective

Cuicui Luo, Luis Seco,

Department of Mathematics, University of Toronto, Toronto, M5S 2E4 Ontario, Canada

***Abstract*—Risk analytics has been popularized by some of today's most successful companies through new theories such as enterprise risk management. Maximizing the benefit from investments on projects can be more based on the correlation structure dynamically from various different sources. It becomes important to assess the forecasting performance of the stochastic correlation models to achieve higher predictive power for risk analytics. We conduct evaluation of stochastic correlation modeling in risk analytics from a financial perspective in this paper. We compare the out-of-sample forecasting performance of exponentially weighted moving average model(EWMA) of Riskmetrics, the Dynamic Conditional Correlation Multivariate GARCH (DCC), the orthogonal GARCH(OGARCH) and the generalized orthogonal GARCH (GOGARCH)using daily returns data from various markets. We find that OGARCH has the best performance and all the multivariate GARCH models outperform EWMA model in risk measurement.**

Keywords: risk management, stochastic correlation, forecasting.

I. Introduction

Risk analytics has been popularized by some of today's most successful companies through new theories such as enterprise risk management. It drives business performance using new sources of data information and advanced modeling tools and techniques. For example, underwriting decisions in the electric power, oil, natural-gas, and basic-materials industries can be improved by advanced credit-risk analytics so that higher revenues and lower costs are yielded through analytics of their commodity exposures. By incorporating, we help clients produce models with significantly higher predictive power.

However, investments from different sources of projects, products and markets can be highly correlated due to the interconnelction among these projects, products and markets. Maximizing the benefit from these investments cannot be based on the data and models individually from different sources; it may be more based on the correlation structure dynamically from different sources. For example, value at risk (VaR) has been widely used in financial institutions as a risk management tool after its adoption by the Basel Committee on Banking(1996). Modeling time-varying volatility and correlation for portfolios with a large number of assets is critical and especially valuable in VaR measurement. Significantly higher predictive power has been observed when considering correlation structure in VaR modeling.

Many multivariate stochastic correlation models have been proposed to model the time-varying covariance and correlations. Following the great success of univariate GARCH model in modeling the volatility, a number of multivariate GARCH models have been developed(see [1],[2] and [3]). The Dynamic Conditional Correlation Multivariate GARCH [4] has been used widely to model the stochastic correlation in energy and commodity market([5], [6],[7]). The correlation in crude oil and natural gas markets has been modeled by the orthogonal GARCH in [8] and the generalized orthogonal GARCH [9] model is also developed. Besides the multivariate GARCH models, the exponentially weighted moving average model (EWMA) of [10] uses to forecast variances and covariances, which is the simplest matrix generalization of a univariate volatility mode.

Since so many models have been developed, the prediction accuracy of the time-varying correlations becomes a major concerns in time series data mining. A number of studies have compared the forecasting performance of the multivariate correlation modes. In [11], it shows that the DCC model outperforms other alternatives in modeling time-varying covariance. It is noted that The optimal hedge fund portfolio constructed by dynamic covariance models has lower risk [12] . [13] provide further evidence that the use of multivariate GARCH models in optimal portfolios selection has better performances than static models and also show that exponentially weighted moving average(EWMA) model has the best performance with superior risk-return trade-off and lower tail risk. [14] compare the performance of some Large-scale multivariate GARCH models using over 50 assets and find that there is value in modeling time-varying covariance of large portfolio by these models. [15] also find that the multivariate GARCH models provide a substantial improvement to the forecast accuracy of the time-varying correlation. The out-of-sample forecasting accuracy of a range of multivariate GARCH models with a focus on large scale problems is also studied by [16] and [17].

This paper aims to evaluate the forecasting performance of RiskMetrics EWMA, DCC, OGARCH and GOGARCH models for the correlation between S& P 500 index and US Generic Government 10 year yield bond index over 10 years period from 2002 to 2013. First we estimate these model and obtain out-of-sample forecasts of time-varying correlations.Then mean absolute error (MAE) and Confidence Set(MCS) approach are applied to assess the prediction abilities. We also compute one-step-ahead out-of-sample VaR of an equally weighted portfolio and perform a backtesting analysis.

The paper proceeds as follows. Section 2 introduces stochastic correlation models namely, the RiskMetrics EWMA model, DCC, OGARCH and GOGARCH. Section 3 presents the evaluation measures used to compare the forecast performance of different models. Section 4 explains the data involved and presents empirical results on the forecast comparison and section 5 concludes.

II. Stochastic Correlation Models

There are many methods to estimate the covariance matrix of a portfolio. In this paper, we compare the forecasting performance of the methods that are widely adopted by market practitioners. In this section, we review these stochastic correlation models.

Let y_t be a k $\times 1$ vector multivariate time series of daily log returns on k assets with length T :

$$y_t = \mu_t + \epsilon_t$$

$$E(y_t|\Omega_{t-1}) = \mu_t$$

$$Var(y_t|\Omega_{t-1}) = E(\epsilon_t \epsilon_t'|\Omega_{t-1}) = H_t$$

Where Ω_{t-1} denote sigma field generated by the past information until time t-1.

A. *Riskmetrics EWMA*

The exponentially weighted moving average(EWMA) models are very popular among market practitioners. The RiskMetrics EWMA model assigns the highest weight to the latest observations and the least to the oldest observations in the volatility estimate.

The multivariate form of EWMA model is defined as

$$H_t = \lambda H_{t-1} + (1-\lambda) y_{t-1} y_{t-1}'$$

For each individual element, it is given by

$$\sigma_i^2, j, t = \lambda\sigma_{i,t-1}\sigma_{j,t-1} + (1-\lambda)y_{i,t-1}y_{j,t-1}$$

λ is the decay factor, which determines the importance of historical observations used for estimating the covariance matrix. The value of the decay factor depends on sample size and varies by asset class. **(alias?)**(1996) suggest that a decay factor of 0.94 is used for daily data set.

Given the decay factor and initial value $\hat{\Sigma_0}$, it is very easy to forecast the conditional covariance matrix. $\hat{\Sigma_0}$ is usually the full sample covariance matrix,which is defined as

$$\hat{\Sigma_0} = \frac{1}{T-1}\sum_{t=1}^{T}(y_t - \bar{y})'(y_t - \bar{y})$$

The EWMA model is very easy to implement for a larger number of assets and the conditional covariance is always semi-definite. The forecast of conditional covariance is also straightforward. The downside of this model is a lack of firm statistical basis and has to estimate decay factor.

B. Multivariate GARCH Models

Following the great success of univariate GARCH model in modeling the volatility, a number of multivariate GARCH models have been proposed for conditional covariance and correlation modeling. The first direct extension of univariate Garch model is VEC-GARCH model of [1], and BEKK models [2]. When there ia a big data sets in the portfolio, the large number of covariance is difficult to estimate and evaluation is also complicated. The large number of unknown parameters in these models prevent their successful application in practice.

To solve the larger-scale problems, factor and orthogonal models are introduced. [19] introduced Constant Conditional Correlation (CCC) model with the restriction of constant conditional correlation which reduces the number of parameters. By making the conditional correlation matrix time-dependent, CCC is generalized by the Dynamic Conditional Correlation (DCC) model [4]. DCC model captures the advantages of GARCH models and simplifies the computation of the multivariate GARCH.

In the DCC model, time series of daily log returns on k assets y_t is conditionally multivariate normal with mean zero. That is,

$$y_t = \epsilon_t$$

$$H_t = D_t R_t D_t$$

$$D_t = diag(\sigma_{11,t}, \ldots.., \sigma_{kk,t})$$

wher eH_t is the conditional covariance matrix. D_t is a diagonal matrix with $\sigma_{11,t}$,…,$\sigma_{kk,t}$ on main diagonal, which can be estimated form the univariate GARCH models.

As noted by [4], R_t also is the conditional covariance matrix of the standardized returns. Let standardized residuals z_t be

$$z_t = D_t^{-1}\epsilon_t$$

and

$$E(z_t z_t' | \Omega_{t-1}) = R_t$$

In this paper, we consider DCC GARCH(1,1) models by [4]. That is, the dynamics for the conditional correlations and the conditional variances follow GARCH-type model. The DCC model can be estimated in two steps. The first step is to estimate the conditional volatility,$\sigma_{kk,t}$ by using GARCH(1,1) model

$$\sigma_{kk,t}^2 = \omega_k + \alpha_k\epsilon_{k,t-1}^2 + \beta_k\sigma_{kk,t-1}^2$$

The second step is to estimate the time varying stochastic correlationR_t.

$$Q_t = (1 - \alpha_0 - \beta_0)\bar{Q} + \alpha_0 z_{t-1}z_{t-1}' + \beta_0 Q_t - 1$$

$$R_t = diag(Q_t)^{-1} Q_t diag(Q_t)^{-1}$$

where $\bar{Q}$ is the unconditional correlation matrix of z_t

The advantage of DCC model is that it separates the estimation of the volatility of each time series (with great flexibility, using single univariate models) and the correlation part (with the strong constraint imposing the same dynamics to all the correlations). The problem is that α_0 andβ_0 in DCC are scalars, so that all the conditional correlations follow the same dynamics.

C. Orthogonal GARCH Model

In addition to the DCC model, the orthogonal GARCH (OGARCH) model was introduced by [20] and [21] based on univariate GARCH model and principal component analysis(PCA). The OGARCH model is computationally simpler than the multivariate GARCH models for a large dimensional covariance matrix because the large number parameters is reduced by PCA. OGARCH has achieved outstanding accuracy in forecasting correlation.

In the orthogonal GARCH model, the observed time series are transformed to uncorrelated time series by using the principal component analysis.

Let y_t be a multivariate time series of of zero mean daily returns on k assets with length T, then the T $\times k$ matrix Y can be approximated by

$$Y = PW'$$

Where the T $\times k$ matrix P is the first n principle components of the covariance matrix of Y for n $\leq$ k. W is a k $\times k$ orthogonal matrix of the eigenvectors of arranged in descending order of the corresponding eigenvalues.

Then we can obtain

$$H_t = W\Sigma_t W'$$

In this paper, the conditional variance of the ith principal component p_i, i = 1,...,N, is modeled by a GARCH(1,1) model as

$$p_{i,t} = \mu_{i,t} + \epsilon_{i,t}$$

$$\sigma_{i,i,t}^2 = \omega_i + \alpha_i {\epsilon_{i,t-1}}^2 + \beta_i \sigma_{i,i,t-1}^2$$

The OGARCH models reduce unknown parameters significantly and is widely used, but it does not work well when the correlation of the time series is not strong.

D. Generalized Orthogonal GARCH Model

The GOGARCH model was first proposed by [9] as a natural generalization of the orthogonal GARCH(OGARCH). In OGARCH model, the matrix is assumed to be orthogonal which only contains a very small subset of all possible invertible matrices, while the orthogonal requirement is relaxed in GOGARCH model. [9] argued that OGARCH often underestimate the correlations because of the orthogonal restriction.

Let y_t be the same T $\times k$ time series as defined in OGARCH model and y_t is transformed to a linear combination of n uncorrelated factors ϵ_t as:

$$y_t = A\epsilon_t$$

Where A is $k \times k$ non- singular matrix. ϵ_t have unconditional unit variance Σ and $\Sigma = Var(\epsilon_t) = I_n$. In this paper, we assume that each unobserved factor ϵ_t follows Garch(1,1) model:

$$\sigma_{i,i,t}^2 = (1 - \alpha_i - \beta_i) + \alpha_i {\epsilon_{i,t-1}}^2 + \beta_i \sigma_{i,i,t-1}^2$$

$$\Sigma_t = diag(\sigma_{1,1,t}^2, ..., \sigma_{n,n,t}^2)$$

Then the conditional covariance matrix of y_t is given by

$$H_t = A\Sigma_t A'$$

And the unconditional covariance matrix H of y_t is defined as $H = A\Sigma A = AA'$. Let P and Λ be orthonormal eigenvectors of H and a diagonal matrix with the corresponding eigenvalues respectively. Then A is decomposed by

$$Z = P\Lambda^{1/2}L'$$

The parameters can be estimated by two-step approach proposed by [9]. First, H is estimated H by sample variance of y_t and Λ can be estimated after the estimation of H. Then L, α and βcan be estimated by maximizing a multivariate likelihood function.

III. Model Evaluation Measures

It is very difficult to evaluate the performance of different models because different investors have different concerns. Risk management mangers care more about the extreme returns and volatility, while portfolio managers may pay more attention on the influence of the correlation on these returns.

Suppose the return of a portfolio return $y_{p,t}$ at time t is given by

$$y_{p,t} = \mu_{p,t} + \epsilon_{p,t}$$

$$\mu_{p,t} = \sum_{i=1}^{n} \omega_{i,t}\mu_{i,t}$$

$$\sigma^2_{p,t} = \omega H_t \omega'$$

Wher e$\mu_{p,t}$, $\sigma^2_{p,t}$ and $\epsilon_{p,t}$ are conditional mean and variance of the portfolio at time t respectively. H_t is conditional covariance matrix defined in section 2. $\omega_{i,t}$ is the portfolio weight of asset i at time t and $\sum_{i=1}^{n} \omega_{i,t} = 1$.

We assume that the conditional distribution is Gaussian, then the portfolio returns are also normal because the multivariate normal distribution are closed under linear transformations ; see [22], [23]. The portfolio Value-at-Risk (VaR) for 1 day horizon at a α confidence level is

$$VaR_{t+1} = \mu_{p,t} + z_\alpha \sigma_{p,t}$$

Where z_α denotes is the critical value of the corresponding quantile α. In this paper, we compute a 95% VaR using normal distribution with $z_\alpha = -1.65$ and we assume $\mu_{p,t}$ is constant and estimated by the sample mean of the portfolio return [24]. A number of papers [25] suggest that equally weighted portfolio strategy(called ”1/N”) consistently outperforms almost other optimization strategies. Therefore, we uses an equally weighted portfolio in our analysis.

A. *Mean Absolute Error*

To evaluate the out of sample forecasting ability of different multivariate models, we first calculate some popular statistical loss functions. The loss functions we choose to assess the performance of competing models in volatilities forecasting is mean absolute error(MAE).

The forms of MAE is given as follows:

$$MAE = \frac{1}{T}\sum_{t=1}^{T}\left|\hat{\sigma_t^2} - h_t\right|$$

Where h_t and $\hat{\sigma_t}$ are variance forecast and actual volatility at time t respectively. Since the actual volatility σ is unobservable, we substitute of squared return y_t^2 to the actual conditional variance (see [26] and [27]).

B. *Model Confidence Set Approach*

In oder to select an optimal model with superior predictive ability in out of sample forecasting among these different multivariate models , we consider Model Confidence Set(MCS) approach,the test introduced by [28]. The advantage of the MCS is that it performs a joint comparison across a full set of candidate models and does not specify a benchmark model.

Let us denote by $d_{i,j,t}$ the loss deferential between models i and j at time t and $L_{i,t}$ is a loss function for model i at time t, then the null hypothesis is given by

$$H_0\colon E[d_{i,j,t}] = E[L_{i,t} - L_{j,t}] = 0, \forall j, i \in M$$

Where $M \subset M_0$ and the starting set M_0 contains all the models.

The initial step sets M = M0. MCS performs an iterative selection procedure. Given a confidence level α, the model with the worst performance is removed from the set at step k if the null is rejected . Repeat this procedure until the null hypothesis cannot be rejected at given significance level. In order to test H_0, we used the following t-statistic $t_{i,j}$:

$$t_{i,j} = \frac{\bar{d_{i,j}}}{\sqrt{Var\hat{(}\ \bar{d}_{i,j})}}$$

$$\bar{d_{i,j}} = \frac{1}{T}\sum_{t=1}^{T} \bar{d_{i,j,t}}$$

where $Var\hat{(}\ \bar{d}_{i,j})$ is estimate of the variance of average loss differential. p value of the test statistics and $Var(\bar{d_{i,j}})$ are determined by using a bootstrap approach.

In this paper, we use the range test statistics $t_R = max_{i,j\in M}|t_{i,j}|$ introduced by [28], because it involves the fewest pairwise comparisons. The elimination rule for the range statistics is $e_M = argmax_{j\in M} sup_{i\in M} t_{i,j}$. The entire procedure continues to repeat on the smaller set of models until the null hypothesis is not rejected. In our analysis , we will use the following MAE based loss functions [29] :

$$L_{mae} = \left|\hat{\sigma_t^2} - h_t\right|$$

Where the squared return is considered an unbiased proxy of the actual conditional variance.

C. Backtesing

Value at Risk (VaR) is widely used as a measure of risk in portfolio risk management . When we consider conditional stochastic correlations, testing the conditional accuracy of the performance in forecasting becomes important.In order to analyze our results implied by different time-varying volatilities models, we compute one-day ahead out-of-sample VaR and perform a backtesting analysis.

The conditional coverage test proposed by [30] is a method to test if the VaR violations are independent and the average number of violations is correct conditional coverage. It is a combination of unconditional test and independence of the violations test. The sequence of $VaR_{t,m}$ violations for model m are defined by an indicator function:

$$I_{t,m} = \begin{cases} 1 & \text{if } y_t < -VaR_{t,m}, \\ 0 & \text{if else.} \end{cases}$$

We start by introducing a simple unconditional test for the average probability of a VaR violation. The null hypothesis of correct unconditional coverage test is that $\pi = \alpha$, where π is coverage rate in a particular model. Suppose there are T_1 days in total with $I_{t,m} = 1$ in a sample period of T, then we get the likelihood function

$$L(\pi) = \pi^{T_1}(1-\pi)^{T-T_1}$$

$$L(\alpha) = \alpha^{T_1} 1 - \alpha^{T-T_1}$$

By the likelihood ratio test we get

$$LR_{uc} = -2log(L(\alpha)/L(\pi)) \sim \chi^2$$

Under the independence hypothesis, it is assumed that the violation sequence is described as a first order Markov sequence with transition probability matrix

$$\pi = \begin{bmatrix} \pi_{00} & \pi_{01} \\ \pi_{10} & \pi_{11} \end{bmatrix}$$

where $\pi_{ij} = P(I_{t+1} = j|I_t = i) for$ i,j=0,1. Note that $1 - \pi_{00} = \pi_{01}$ and $1 - \pi_{10} = \pi_{11}$ The null hypothesis independence test is assumed that $\pi_{01} = \pi_{11} = \pi$. The likelihood function for a sample of T observations is

$$L(\pi_1) = {\pi_{01}}^{T_{01}}(1 - \pi_{01})^{T_{00}}{\pi_{11}}^{T_{11}}(1 - \pi_{11})^{T_{10}}$$

where $T_{ij}, i, j = 0, 1$ is the number of observations with a j following an i. Then the independence hypothesis is tested by using a likelihood ratio test

$$LR_{ind} = -2log(L(\pi)/L(\pi_1)) \sim \chi^2$$

Finally, the likelihood ratio of conditional coverage test is $L_{cc} = LR_{uc} + LR_{ind}$

IV. Empirical Results

The data used for the test are the daily close price to S&P 500 index and US Generic Government 10 Year Yield bond index.The sample period used here is Sep 9, 2002 through Sep 9, 2013, for a total of 2549 daily observations. We remove common holidays and weekends across these time series to minimize the possibility of inducing spurious correlation. In order to test the robust of the results, the models are applied to two subsample periods. The first period is financial crisis period (2008.4-2009.4) and the second is normal period (2012.10-2013.10). The historical daily returns of S&P 500 and bond index are shown in Figure 1.

First we estimate the dynamic of the conditional correlation for the full sample by using Riskmetircs EWMA, OGARCH,GOGARCH and DCC Garch models. We compare the implied conditional correlation in Figure 2. The figure shows that the estimated correlation of GOGARCH is quite stable which is around 0.5. However, the correlations estimated by the others are very volatile and EWMA model gives the most volatile correlation curve, The correlations under GOGARCH model have never dropped to 0.4 based on our calculation, even when the col-

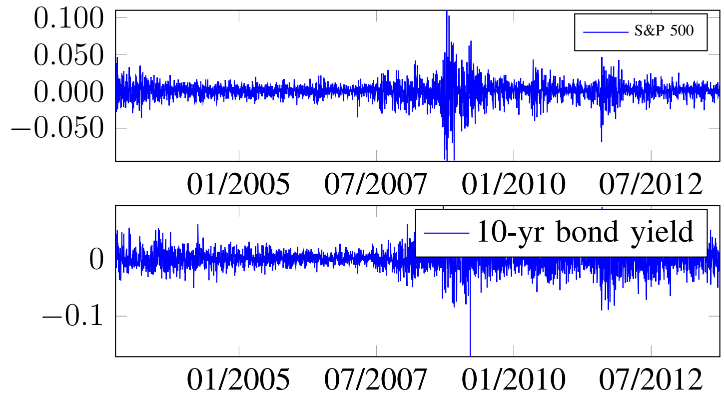

Fig. 1. Daily return of S&P 500 and 10-yr Bond Index in 2002-2012

orations estimated by the other model are negative. Furthermore, the correlation estimated by EWMA, OGARCH and DCC Garch models all increased significantly around 2008 except GOGARCH. A very sparse observation appears at the end of 2005 in Figure 1 and a peak also appears in the conditional correlation estimated by GOGARCH model .

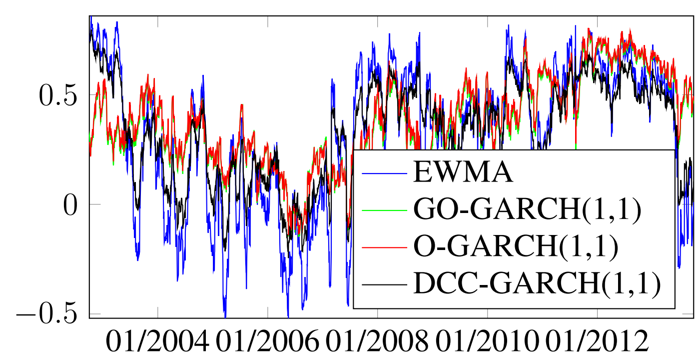

Fig. 2. In sample estimation of stochastic correlation between S&P 500 and 10-yr Bond Index ,2002-2013

Now let us compare these models by investigating the forecasting performance of the 95% VaR for an equally weighted portfolio by using MAE, MCS and conditional coverage test. The confidence level for the MCS approach we use is 0.05. We also set the number of bootstrap samples l to 10,000 in order to obtain the distribution under the null. The critical value of LR_cc statistic with two degrees of freedom used throughout the backtesting process is 5.99 at 95% confidence level. [31] suggested that 95% confidence level fits well for backtesting purposes. We construct MCS test at 10% confidence level using range statistics.

Table 1 illustrates some test statistics of conditional covariance estimated for in-sample prediction by EWMA, DCC GARCH(1,1), OGARCH(1,1) and GOGRACH(1,1). The best performance in forecasting under the in-sample analysis is OGARCH with the least MAE. GOGARCH has largest MAE because of the flat curve in Figure 2. The 10% MCS consist solely of OGARCH model, and all theses models except EWMA fail the conditional coverage test for 95% VaR. OGARCH(1,1) outperform the other models according to all evaluation criteria by in sample analysis.

TABLE I
TEST STATISTICS FOR IN SAMPLE ESTIMATION FOR THE WHOLE PERIOD

	MAE	MCS	LR_{cc}(0.95)
EWMA	4.13E-05	0.0096	0.945
DCC	4.10E-05	0.0834	5.805
OGARCH	3.62E-05	1	Inf
GOGARCH	4.22E-05	0.0834	10.6589

A. First Subsample Period

In order to examine the time-varying nature of the conditional correlations, as well as to investigate the forecasting performance of these models in a turbulent period, we split the data sample into 2 subsamples. In the first subsample, we only consider the first 1777 observations in order to examine the performance of the models during the 2008 financial crisis period. We split the subsample into two parts, 5.5-year estimation period which is used for in sample estimation and the subsequent 1.5-year out of sample forecast periods. We use a rolling window size of 1413 days to forecast the conditional covariance in financial crisis.

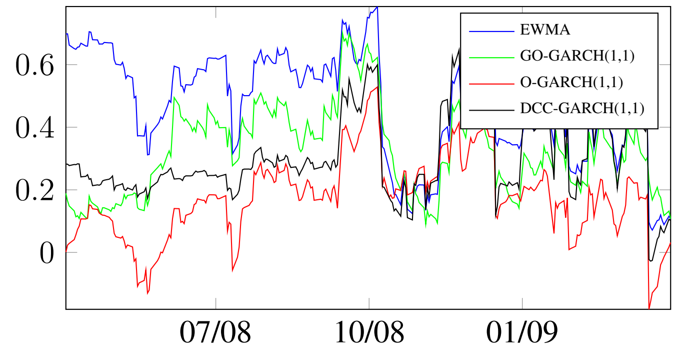

Fig. 3. 1-day ahead out-of-sample conditional correlation forecast between S&P 500 and 10-yr Bond Index in 2008-2009

Figure 3 presents the forecasting performance of the four models mentioned above during 2008 financial crisis. Compared with Figure 2, the estimated correlations by GOGARCH is more volatile and the correlation reached peak in Oct. 2008. The correlation curves are very close to each other. The correlation is always positive between April 2008 and April 2009. Table 3 reports some test statistics of out of sample predication performance of the 95% VaR for an equally weighted portfolio by using MAE, MCS and conditional coverage test. All these four models pass the conditional coverage test for 95% VaR during financial crisis period and all these models are included in MSC except EWMA. That means, all these models perform well during crisis. We conclude OGARCH has the best performance based on all the test statistics in this period.

TABLE II
TEST STATISTICS FOR OUT-OF -SAMPLE FORECAST IN 2008-2009

	MAE	MCS	LR_{cc}(0.95)
EWMA	1.04E-04	0.0066	0.016
DCC	9.80E-05	0.0139	1.444
OGARCH	9.61E-05	1	1.119
GOGARCH	9.71E-05	0.0165	1.119

B. Second Subsample Period

In the second subsample, we estimate the parameters by using the 1777 observations in first 7 years and do one step ahead forecasts for the last 3 years with rolling window of 1777 days. We apply the proposed models to obtain out-of-sample one-step-ahead forecasts of the conditional covariance matrix of all assets.

From Figure 5 we can see the curves are overlapping and very close to each other. All the models pass conditional coverage test based on table 4. EWMA and DCC both have very volatile correlations.The 10% MCS consistS OGARCH and GOGARCH. All the test statistics shown in Table 3 suggest that OGARCH and GOGARCH models both perform well and OGARCH achieves the best performance.

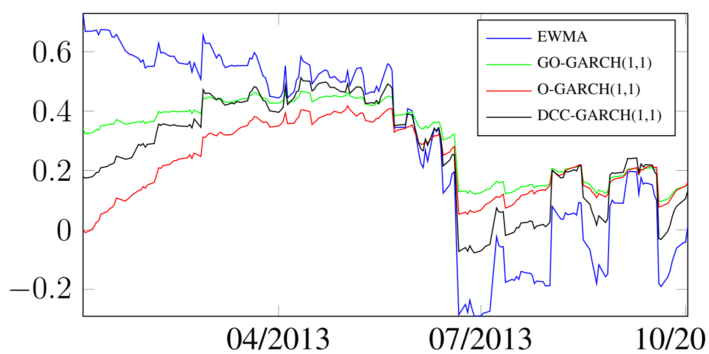

Fig. 4. 1-day ahead out-of-sample conditional correlation forecast between S&P 500 and 10-yr Bond Index in 2013

TABLE III
TEST STATISTICS FOR OUT-OF -SAMPLE FORECAST IN 2013

	MAE	MCS	$LR_{cc}(0.95)$
EWMA	5.11E-05	0	0.0734
DCC	5.09E-05	0.001	1.0371
OGARCH	4.90E-05	1	0.0135
GOGARCH	4.92E-05	0.161	0.2164

V. CONCLUSION

In this paper we investigate the forecasting ability of four multivariate stochastic correlation models. These models are EWMA, DCC, OGARCH and GOGARCH in different market conditions. All theses models have a quite good forecasting performance during 2008 financial crisis period and after 2008 crisis. GOGARCH and OGARCH modelS outperform the others. OGARCH acheives the best performance during this two sub periods. During the stable market conditions, the out of sample conditional correlations are very different among these models. DCC outperforms all the other three models. The out-of-sample sample estimation results show that OGARCH model has the best performance. The overall performance of multivariate Garch models is better than EWMA.

REFERENCES

[1] T. Bollerslev, R. F. Engle, and J. M. Wooldridge, "A capital asset pricing model with time-varying covariances," *The Journal of Political Economy*, pp. 116–131, 1988.

[2] R. F. Engle and K. F. Kroner, "Multivariate simultaneous generalized arch," *Econometric theory*, vol. 11, no. 01, pp. 122–150, 1995.

[3] A. Silvennoinen and T. Teräsvirta, "Multivariate garch models," in *Handbook of Financial Time Series*. Springer, 2009, pp. 201–229.

[4] R. Engle, "Dynamic conditional correlation: A simple class of multivariate generalized autoregressive conditional heteroskedasticity models," *Journal of Business & Economic Statistics*, vol. 20, no. 3, pp. 339–350, 2002.

[5] D. Bicchetti and N. Maystre, "The synchronized and long-lasting structural change on commodity markets: evidence from high frequency data," *Algorithmic Finance*, vol. 2, no. 3, pp. 233–239, 2013.

[6] A. Creti, M. Joëts, and V. Mignon, "On the links between stock and commodity markets' volatility," *Energy Economics*, vol. 37, pp. 16–28, 2013.

[7] Y. Wang and C. Wu, "Forecasting energy market volatility using garch models: Can multivariate models beat univariate models?"

Energy Economics, vol. 34, no. 6, pp. 2167–2181, 2012.

[8] C. Alexander, "Correlation in crude oil and natural gas markets," *Managing energy price risk*, pp. 573–606, 2004.

[9] R. Van der Weide, "Go-garch: a multivariate generalized orthogonal garch model," *Journal of Applied Econometrics*, vol. 17, no. 5, pp. 549–564, 2002.

[10] RiskMetrics, "Technical document," *J.P.Morgan*, Dec. 1996.

[11] A. Wong and P. Vlaar, "Modelling time-varying correlations of financial markets," *De Nederlandsche Bank NV Research Memorandum*, vol. 739, p. 0319, 2003.

[12] D. Giamouridis and I. D. Vrontos, "Hedge fund portfolio construction: A comparison of static and dynamic approaches," *Journal of Banking & Finance*, vol. 31, no. 1, pp. 199–217, 2007.

[13] R. D. Harris and M. Mazibas, "Dynamic hedge fund portfolio construction," *International Review of Financial Analysis*, vol. 19, no. 5, pp. 351–357, 2010.

[14] R. Engle and K. Sheppard, "Evaluating the specification of covariance models for large portfolios," *New York University, working paper*, 2008.

[15] J.-r. Lu, P.-L. Tsai *et al.*, "Forecasting equity market correlation: A comparison of alternative measures," *Journal of International and Global Economic Studies*, vol. 2, no. 2, pp. 77–95, 2010.

[16] M. Caporin and M. McAleer, "Robust ranking of multivariate garch models by problem dimension," *Computational Statistics & Data Analysis*, 2012.

[17] S. Laurent, J. V. Rombouts, and F. Violante, "On the forecasting accuracy of multivariate garch models," *Journal of Applied Econometrics*, vol. 27, no. 6, pp. 934–955, 2012.

[18] J. Morgan, *Riskmetrics: technical document*. Morgan Guaranty Trust Company of New York, 1996.

[19] T. Bollerslev, "Modelling the coherence in short-run nominal exchange rates: a multivariate generalized arch model," *The Review of Economics and Statistics*, pp. 498–505, 1990.

[20] C. Alexander and A. Chibumba, "Multivariate orthogonal factor garch," *University of Sussex*, 1997.

[21] C. Alexander, "A primer on the orthogonal garch model," *Manuscript ISMA Centre, The Business School for Financial Markets, University of Reading*, 2000.

[22] M. H. Pesaran and P. Zaffaroni, "Model averaging and value-at-risk based evaluation of large multi-asset volatility models for risk management," *CEPR Discussion Paper*, 2005.

[23] P. Christoffersen, "Value–at–risk models," in *Handbook of Financial Time Series*. Springer, 2009, pp. 753–766.

[24] A. A. Santos and G. V. Moura, "Dynamic factor multivariate garch model," *Computational Statistics & Data Analysis*, 2012.

[25] Y. Plyakha, R. Uppal, and G. Vilkov, "Why does an equal-weighted portfolio outperform value-and price-weighted portfolios," *Preprint available at http://dx. doi. org/10.2139/ssrn*, vol. 1787045, 2012.

[26] P. Sadorsky, "Modeling and forecasting petroleum futures volatility," *Energy Economics*, vol. 28, no. 4, pp. 467–488, 2006.

[27] Y. Wei, Y. Wang, and D. Huang, "Forecasting crude oil market volatility: Further evidence using garch-class models," *Energy Economics*, vol. 32, no. 6, pp. 1477–1484, 2010.

[28] O. E. Barndorff-Nielsen, P. R. Hansen, and N. Lunde, A.and Shephard, "Realized kernels in practice: Trades and quotes," *The Econometrics Journal*, vol. 12, no. 3, pp. C1–C32,

2009.

[29] A. J. Patton, "Volatility forecast comparison using imperfect volatility proxies," *Journal of Econometrics*, vol. 160, no. 1, pp. 246–256, 2011.

[30] P. F. Christoffersen, "Evaluating interval forecasts," *International economic review*, pp. 841–862, 1998.

[31] P. Jorion, *Financial Risk Manager Handbook 2001-2002*. John Wiley & Sons, 2001, vol. 96.